Mathematics Case Study:

Greg Hawtrey and his Grade 8/10 class

Context

The school

Fish Hoek High School is situated in the municipality of Fish Hoek, on the outskirts of Cape Town. Fish Hoek is a historically white area, and very nearby are the 'suburbs' of Ocean View (mostly Cape Coloured) and Masiphumelele (mostly Black, Xhosa, Malawian, Zimbabwean, with some brick houses and many shacks).

The students at the school are mostly white, but there are sizeable minorities of Cape Coloured, Black and Asian students. For most, but not all, the students, English is the home language.

The school is a government school which charges fees of about R18950, which is about Euro 1200. This is much more than most South Africans can afford, so in effect this excludes many students from applying for the school. The school is in the highest socio-economic quintile in the country, but the reader should be aware that this quintile spans a very wide range of schools.

There were 984 students on roll in 2015 with 54 teaching staff. The school has five grades: Grade 8 (first year of secondary school) to Grade 12. The age of most students is between 13 and 18. For many years, the school only had Grades 10, 11 and 12 and having the younger students represented a significant change for many of the teachers.

In this school, the Grade 8 and 9 students are grouped for teaching in mixed-ability sets. From Grade 10, they are grouped according to their subject choice for the final matriculation (matric) examination at the end of Grade 12.

The school has projection technology in all classrooms, and all classrooms are equipped with a PC for the teacher to use. There are some laptops available for use by the students, and these are sometimes used for teaching mathematics, but mainly in after-school lessons. The school has three computer rooms and Computer Applications Technology is an optional subject for students in the last three years of school. Students in Grade 10, 11 and 12 use calculators in mathematics lessons but at Grade 8 and 9 calculators are not supposed to be used.

Figure 1: Inside the school, big quad

Figure 2: A typical classroom at FHHS

The class

For the first research lesson, a Grade 8 class was used but for the subsequent three lessons a Grade 10 class was used. We provide information about the latter class. This class made up of students who had chosen to study mathematics and physical science for matric. Only students who have previously achieved well in these subjects tend to choose this combination, so informally the class is seen as 'top' set.

The left hand column of the table of marks below (Figure 3) provides the results of examinations, in percentages, at the end of the first term of 2015, which ended on

Term 1	Term 4
100	**100**
98	96
89	76
80	83
90	79
86	82
84	77
85	79
92	74
95	88
94	93
86	87
92	71
97	95

Term 1	Term 4
100	**100**
97	85
76	81
100	97
100	100
84	77
95	96
93	83
90	77
100	99
87	85
92	91
99	92

Figure 3: Student grades

1st April, so before any of their FaSMEd lessons took place. The right hand column gives the students' grades at the end of the school year, December 2015, so after all three FaSMEd lessons had taken place.

The teacher

A pen portrait of the teacher, Greg Hawtrey, is provided below is Section 3. He is pictured here (Figure 4)

Figure 4: Greg Hawtrey

1- Tasks and resources used

For us, a FaSMEd research lesson was usually based on one of the Mathematics Assessment Project (MAP) lessons developed by the University of Nottingham as formative assessment lessons. For many schools in South Africa, for students to use digital technology in the classroom is not possible, and we decided that all schools would use non-digital technology in our research lessons: small cards to be sorted or matched, big versions of the small cards for whole class work and mini whiteboards.

Research lesson 1 (Grade 8 Properties of exponents)

Lesson 1 10th March 2015

Activity 1: The students were given the questions shown in Figure 5 to answer individually.

Properties of Exponents

1. In each of the following questions write the missing exponents on the dotted lines.
Show your reasoning in the spaces provided on the right.

a) $2+2+2+2=2^{\cdots}$

b) $2\times2\times2\times2=2^{\cdots}$

c) $2^{\cdots}\times2^3=2^6$

d) $2^3\times3^3=6^{\cdots}$

e) $4^3=2^{\cdots}$

f) $(3^{\cdots})^3=3^6$

g) $5^6\div5^2=5^{\cdots}$

h) $5^2-3^2=2^{\cdots}$

i) $3^5\div3^{\cdots}=3^{\cdots}=\frac{1}{3}$

2. Write these five numbers in order of size, from smallest to greatest:

6^0 0^6 3^2 2^3 7^{-1}

Smallest				Greatest

Show your reasoning here:

Figure 5: Pre-lesson assessment (Properties of exponents)

Lesson 2: 11th March 2015

Activity 2: Brief discussion of the pre-lesson assessment task.

Activity 3: The teacher displayed the PowerPoint slide 'Powers of Two' (Figure 6) and asked the students to write first the statement A (8 x 4 = 32) in powers of two on their mini whiteboards and then to write the other three statements (B, C and D) also in powers of 2.

Powers of 2

A: $8 \times 4 = 32$
B: $16 \div 8 = 2$
C: $8 \div 16 = ½$
D: $8 \div 8 = 1$

Figure 6: Statements A, B, C and D to be written as powers of 2

Activity 4: The students were asked to work in groups of four to put the cards shown in Figure 7 into groups.

The instructions for what they should do and how they should organise their group work are shown in Figures 8 and 9.

E1 $2^2 \times 3^2$	E2 $3^2 - 2^3$
E3 $2^2 + 2^3$	E4 $2^2 \div 2^3$
E5 $6^8 \div 6^4$	E6 $2^2 - 2^2$
E7 $3^2 + 3^3$	E8 $4^2 \div 2^3$
E9 $2^3 \div 2^{-2}$	E10 $(2^3)^2$
E11 3×2^2	E12 $2^3 \times 2^3$
E13 $5^2 - 3^3$	E14 $(3^2 \times 2^2)^2$

S1 2^1	S2 2^5
S3 $(-2)^1$	S4 2^{-1}
S5 2^0	S6 2^6
S7 6^4	S8 6^2
S9 0^2	S10 4^3

Figure 7: Two sets of cards to put into groups

What you should do

1. Work out the value of the cards using the laws of exponents or by calculation.
2. Put cards with the same value in groups.
3. Each group should have AT LEAST one card of each colour.
4. Use the blank pink card where necessary.
5. You should end up with ten groups of cards.

Figure 8: What students should do

Working Together

Take turns to:

1. Select an expression card and find **all other cards** that have the same value as the one you have chosen.
2. Explain your matching to your partner.
3. Your partner must check your matching and challenge your explanation if they disagree.
4. Once agreed, glue the cards onto the poster and record your explanation for each match.
5. Continue to take turns until you have ten groups of cards.

Figure 9: How to work in a group

Activity 5: Students shared work, see instructions in Figure 10.

Sharing Work

One person in your group:

- Write down your card matches on your mini-whiteboard.
- Go to another group's desk and compare your work with theirs.
- If there are differences in your matches, ask for an explanation.
- If you still don't agree, explain your own thinking.

The other person(s) should:

- Stay at your desk, and be ready to explain the reasons for your group's decisions.

Figure 10: Instructions for sharing work

Activity 4: Whole class discussion: going through the answers

Lesson 3: 12th March 2015

Activity 5: Students completed a post-lesson assessment, as shown in Figure 11.

Properties of Exponents (Revisited)

1. In each of the following questions write the missing exponents on the dotted lines. Show your reasoning in the spaces provided on the right.

a) $2 \times 2 \times 2 = 2^{\cdots}$

b) $3 + 3 + 3 = 3^{\cdots}$

c) $6^{\cdots} \times 6^4 = 6^6$

d) $3^3 \times 4^3 = 12^{\cdots}$

e) $4^5 = 2^{\cdots}$

f) $(6^{\cdots})^4 = 6^8$

g) $10^6 \div 10^3 = 10^{\cdots}$

h) $10^2 - 6^2 = 4^{\cdots}$

i) $4^5 \div 4^{\cdots} = 4^{\cdots} = \frac{1}{16}$

2. Write these five numbers in order of size, from greatest to smallest:

11^{-1} 10^0 0^{10} 5^2 2^5

Greatest				Smallest

Show your reasoning here:

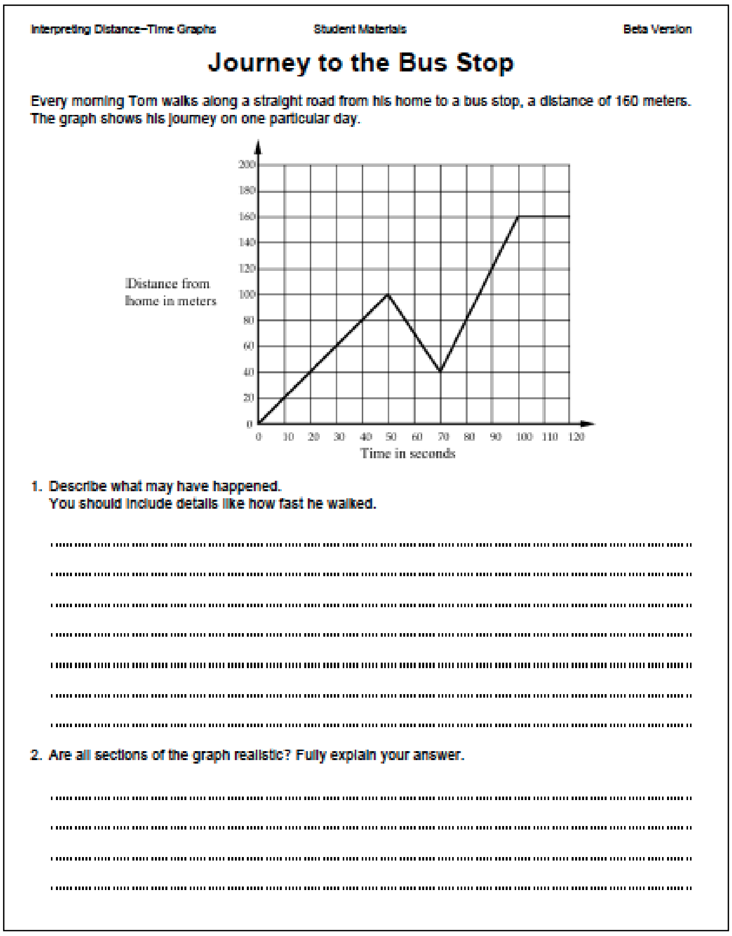
Figure 11: Post-lesson assessment

Research lesson 2 (Grade 10, time-distance graphs)

After Research Lesson 1, Greg switched to using his Grade 10 class.

Lesson 1: 15th April 2015

Activity 1: Pre-lesson assessment, see Figure 12.



Interpreting Distance–Time Graphs   Student Materials   Beta Version

Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

1. Describe what may have happened.
   You should include details like how fast he walked.

   ....

   ....

   ....

   ....

   ....

   ....

   ....

2. Are all sections of the graph realistic? Fully explain your answer.

   ....

   ....

   ....

   ....

Figure 12: Pre-lesson assessment (time distance graphs)

Lesson 2: 16th April 2015

Activity 2: Class discussion: choose the story to match the graph (Figure 13).

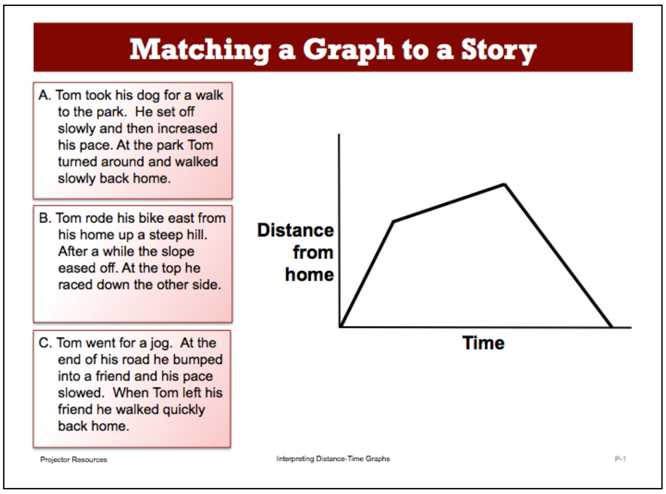


Figure 13: Choose the story to match the graph

Activity 3: The students were asked to work in groups of four to match the cards shown in Figures 14 and 15.

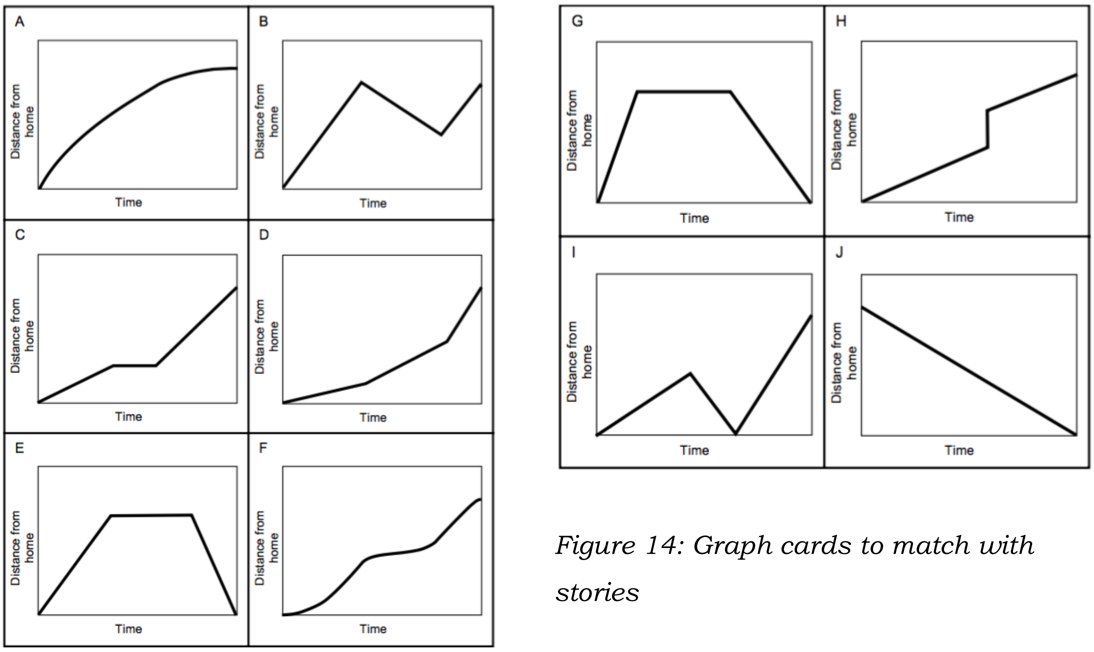


Figure 14: Graph cards to match with stories

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	**2** Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	**4** Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	**6** Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	**8** This graph is just plain wrong. How can Tom be in two places at once?
9 After the party, Tom walked slowly all the way home.	**10** Make up your own story!

Figure 15: Description (story) cards to match

Activity 4: Students shared work, see instructions in Figure 16.

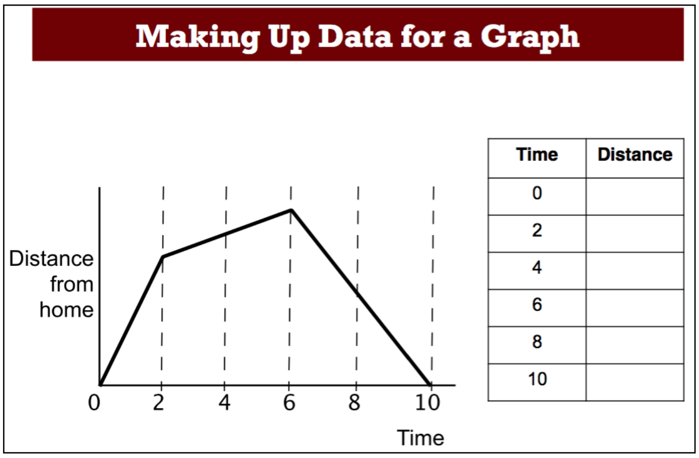
Sharing Work

- One student from each group is to visit another group's poster.
- If you are staying at your desk, be ready to explain the reasons for your group's matches.
- If you are visiting another group:
  - Write your card placements on a piece of paper.
  - Go to another group's desk and check to see which matches are different from your own.
  - If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.
  - When you return to your own desk, you need to consider as a group whether to make any changes to your own poster.

Figure 16: Instructions for sharing work

Activity 5: Institutionalisation, reading out the answers.

Activity 6: Students worked individually to make up data for a graph, see Figure 17.



Time	Distance
0	
2	
4	
6	
8	
10	

Figure 17: Making up data for a graph

Activity 7: Students worked in the same small groups to match a third card set (Figure 18) with the existing card pairs.

P

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

Q

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

R

Time	Distance
0	0
1	18
2	36
3	54
3	84
5	120

S

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

T

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

U

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

V

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

W

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

X

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

Y Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Z Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Figure 18: Tables of values to match with graphs and descriptions

Activity 8: Students completed a questionnaire about the lesson (see Figure 19)

Your teacher's name: Date:

About the FaSMEd lesson: **circle three phrases or words** that stand out for you most.

Difficult Made us discuss Boring Group work Easy

Noisy Confusing Messy Learning from others

Different Made me think Exciting

The FaSMEd lesson was **different** because…

The FaSMEd lesson **would be better** if…

Figure 19: Student questionnaire

Research lesson 3 (Grade 10, Representations of functions and non-functions)

Lesson 1: 26th May 2015

Activity 1: The teachers modelled the activity using big (A4) cards made for the purpose (see Figure 20).

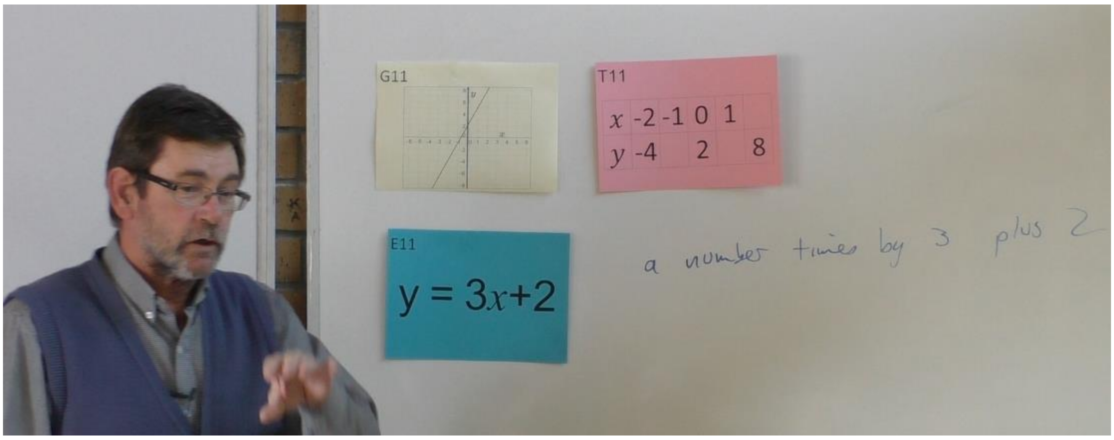


Figure 20: Big cards used for modelling

Activity 2: Students were given four sets of cards: equations, graphs, tables of values and descriptions in words (Figures 21 and 22). They were asked to put them into groups.

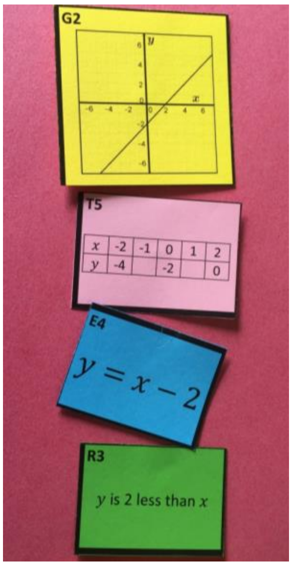
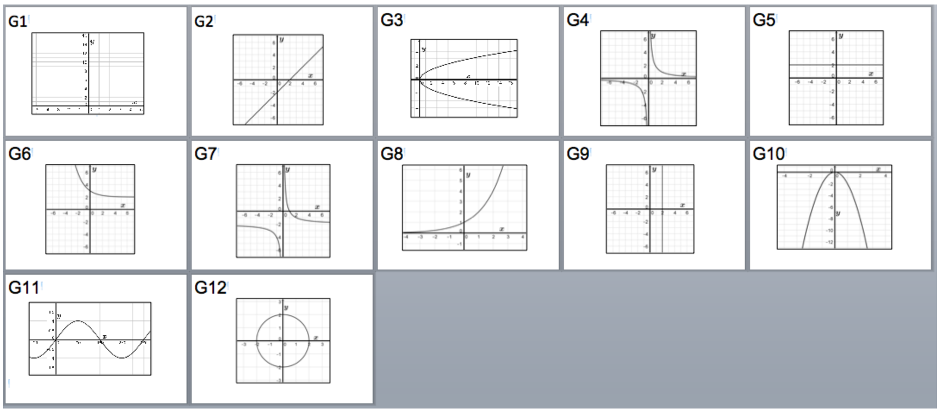


Figure 21: One set of cards, matched

E1 $xy = 2$	E2 $y = 2^{-x} + 2$	E3 $y = x^2$	E4 $y = x - 2$	E5 $y = 2$
E6 $x = 2$	E7 $y = \frac{2}{x} - 2$	E8 $y = 2^x$	E9 $x^2 + y^2 = 2^2$	E10 $y = \sin x$
E	E			

R1 y is the same as -2 multiplied by x multiplied by x	R2 y is the same as2 plus 2 to the power of $-x$	R3 y is 2 less than x	R4 x is the same as y multiplied by y
R5 x multiplied by y is equal to 2	R6 y is constant	R7 y is the same as x multiplied by x	R8 y is 2 less than 2 divided by x
R9 y is the same as the sin of x	R10 The square of x plus the square of y is constant	R	R



T5

x	-2	-1	0	1	2
y	-4		2		0

T10

x	-2	-1		1	
y		1	0		4

T3

x	0	1	4	9	16
y	0	±1	±2	±3	±4

T8

x	-2	-1	0	1	2
y		-2	$\pm\infty$		1

T1

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

T6

x	-2	-1	0	1	
y					-8

T7

x					
y	-2	-1	0	1	2

T2

x	-2	-1	0	1	2
y	6	4	3	$\frac{5}{2}$	$\frac{9}{4}$

T4

x	-2	-1	0	1	2
y	-3		$\pm\infty$		-1

T9

x					
y					

T11

x	-90	0		180	270
y	-1		1	0	

T12

x	-2	-1	0	1	2
y	0	$\pm\sqrt{3}$		$\pm\sqrt{3}$	0

Figure 22: Card sets for students to match. Big A4 versions of these cards were provided for Activity 3 (below).

Lesson 2: 27th May 2015

Activity 3: Students who had completed the activity were asked to go to the board, and to select a set of big cards and paste them on the board. See below, Figure 23, for a photograph.

Figure 23: Pasting big cards on the board

Activity 4: Whole class discussion. The teacher went through the card matches.

Lesson 3: Some time after 26th May

Activity 5: Individual work. The students were asked to fill in the recording sheet (Figure 24) and stick it into their books.

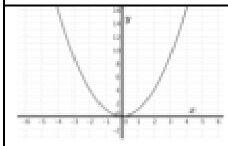
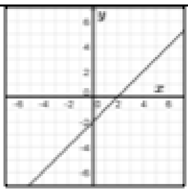
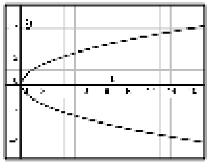
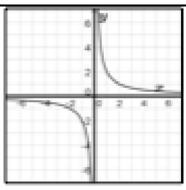
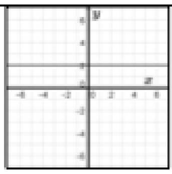
Graph	Equation	Rule	Table
			x y
			x y
			x y
			x y
			x y

Figure 24: Sheet for students to record their answers

Activity 6: Students completed a questionnaire about the FaSMEd lesson. This was almost the same as the one used previously, see Figure 19 above.

Research lesson 4 (Grade 10, Reading functions)

Lesson 1: 3rd September 2015

Activity 1: Brief introduction; the researchers told the students what they should do.

Activity 2: The students worked in pairs to match statement cards with graph cards (Figure 25). Each student was given a mini-whiteboard for rough working or making notes.

S1 $g(x)$ is increasing only for $x > 0$	S2 $g(x) > f(x)$ when x is positive
S3 $f(x)$ is constant	S4 $g(x)$ is always bigger than $f(x)$
S5 $g(x) < f(x)$ when x is positive and $f(x)$ is only positive for $x > 4$	S6 $f(x) > g(x)$ for $0 < x < 5$ and $g(x)$ is negative for $0 < x < 4$
S7 $g(x)$ is positive for $1 < x < 5$	S8 $f(x).\ g(x)$ is positive for $0 < x < 4$
S9 $f(x)$ is increasing from $x = 2{,}5$	S10 $f(x)$ increases when $g(x)$ decreases
S11 $g(x) - f(x) = 9$ when x is 2	S12 $g(x)$ is decreasing only when x is negative
S13 $g(x)$ is always positive	S14 $f(x)$ is negative when x is between 0 and 5
S15 $g(x)$ is always increasing	S16 $f(x) \div g(x)$ is negative for $0 < x < 4$
S17 $f(x) < 0$ for $x < -1$ and for $1 < x < 5$	S18 $f(x).\ g(x) > 0$ when x is between 0 and 1
S19 $g(x) - 2 = f(x)$ when x is -1	S20 $g(x)$ is positive for $0 < x < 4$

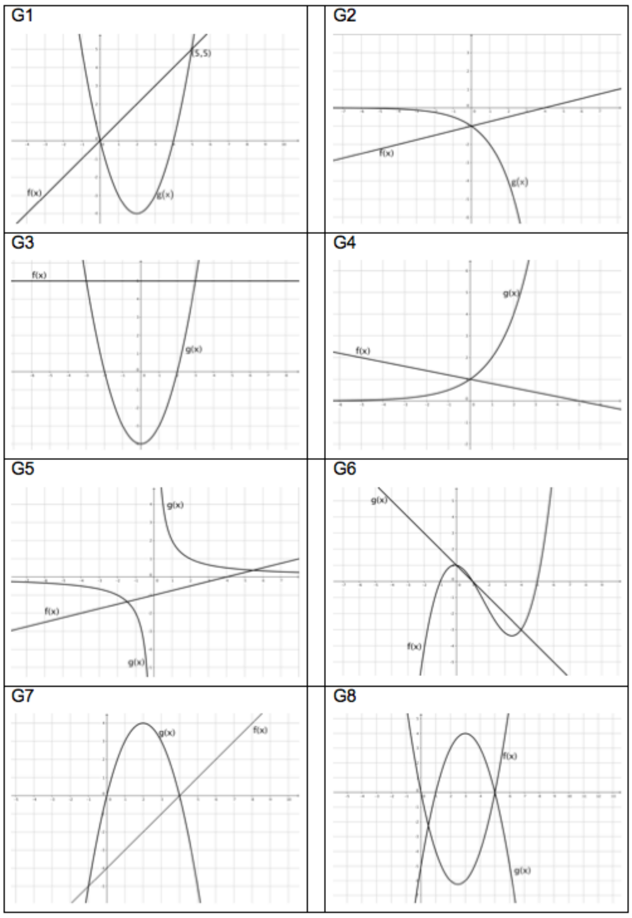


Figure 25: Statement and graph cards to match (small cards for the students to use and big A4 versions for the class discussion)

Lesson 2: 4th September 2015

Activity 3: Students continued to match cards

Activity 4: The teacher had pasted all the big graph cards along the top of the board. He handed out two or three big statement cards to each group and asked them to paste them on the board in under the appropriate graph card. Figure 26 shows the matched cards on the board.

Figure 26: Big cards pasted on the board

Activity 5: Whole class discussion, the phase of institutionalisation as the teacher went through the card matches.

Activity 6: Whole class discussion led by the researchers about the design of the lesson.

Technology used

Sets of small cards for use by the students in group work

Big cards for modelling the activity

Sets of big cards for the institutionalisation phase

Mini whiteboards

Formative assessment

Teacher: Questioning during whole class discussion

Teacher: Pre-lesson assessments (Research lessons 1 and 2); these allowed the teacher to assess prior understanding and plan accordingly.

Teacher: Gathering information from the card matching or sorting activities and intervening accordingly, usually with questions.

Teacher: Gathering information from what the students have written on the mini whiteboards and intervening accordingly, usually with questions.

Students: Gathering information about their peers' understanding from the card matching or sorting activities and acting accordingly, to explain their own reasoning, for example.

2- "Work with teachers"

How was the overall work/sessions with teachers conducted?

Overview

The work of FaSMeD in South Africa was organised by the researchers at AIMSSEC. The research team began working with teachers in the second half of 2014. We did not have research relationships prior to the FaSMEd project. We spent some months recruiting teachers, and made the decision to involve any teachers who chose to take part. In the end we had 20 teachers in 10 schools. Most of these teachers taught three FaSMEd research lessons, one in each of the first three school terms of the academic year beginning in January 2015. For each lesson, we visited the teacher once or twice to plan the lesson, went to school to observe and video record the lesson (often over two days) and then interviewed the teacher. The work with the teachers focused on the planning of lessons and the resources to be used, and included discussion of how formative assessment could be used or was used, with a particular emphasis on the potential of the small and big cards and the mini whiteboards.

Most interactions with the individual teachers were face to face, and interviews were audio- or video-recorded. With some teachers, email was used to make arrangements and send proposed lessons or reports. Other teachers were not regular users of email, and arrangements were often made using WhatsApp.

After each phase of lessons, we held a cluster meeting. At the second and third cluster meetings we asked some teachers to make a short presentation to the group.

This case study is about Greg Hawtrey and his students. Greg's first research lesson was with his Grade 8 class, but he later used his Grade 10 class. Greg teaches at Fish Hoek High School and both he and his colleague, Rob Douglas, took part in the Fasmed research.

In detail

We first met with Greg Hawtrey and Rob Douglas on 3rd December 2014, at their school. We showed them the FaSMEd time-distance graphs activity and explained that we would like them to teach three lessons using similar resources. They both agreed that they would like to take part.

During the school year 2015, we met with Greg as detailed below. In most cases the participants in the meeting were the two researchers, the teacher and, sometimes, his colleague.

- 4th March, to observe an 'ordinary' lesson and to plan the first research lesson. We asked to see an ordinary lesson so that we could get a sense of Greg as a teacher.
- 11th March, to observe and video-record the research lesson (Properties of exponents) and to interview Greg. This lesson was with Greg's Grade 8 class. It was chosen to fit in with the work the class was doing at the time. Greg did the pre- and post-lesson assessments when we were not present.
- 16th April to observe and video-record the second research lesson (Time-distance graphs) and to interview Greg. This lesson was with Greg's Grade 10 class. They did the pre-lesson assessment the previous day. It seemed from the student responses to our questionnaire that this lesson was too easy for this class (see Appendix B).
- 19th May to plan the third research lesson (Multiple representations of algebraic relationships). This lesson was with Greg's Grade 10 class.
- 26th and 27th May, to observe and video-record the third research lesson
- 4th June to interview Greg about the third research lesson
- 8th June to discuss Greg's contribution (Cameo slot) to the second cluster meeting
- 20th August, to plan for the fourth research lesson (Reading function notation)
- 3rd and 4th September to observe and video-record the fourth research lesson, and to interview Greg afterwards
- 27th November, to interview Greg and do the Q-sort with nine of his students

Cluster meetings

Cluster meetings were held three times; at the end of a phase of research lessons. All teachers participating in the research (apart from those too far away: one college with three teachers) were invited to attend the cluster meetings. Not all teachers were able to come to each of the cluster meetings, but many did come. Full notes on the cluster meetings are available in Appendices E, F and G.

Here, below, we provide more details of what happened in each cluster meeting.

Cluster meeting 1: 26th March

This meeting took place at the main AIMS building in Muizenberg, timed to take place after school (3:30 to 5 pm). Fourteen teachers, one visitor (observer) and four colleagues from AIMSSEC attended.

The researchers ran the meeting, first welcoming the teachers and explaining the purpose of the meeting: for teachers to get to know one another and begin to form a 'cluster'; to share what had happened so far, to talk about what happens next in the project and to discuss emerging issues.

Sharing what has happened so far:

- The researchers introduced each of the teachers and explained what they had done.
- The researchers showed a video montage taken from the video recordings of the research lessons from all teachers;
- We distributed a set of research reports to each teacher;
- We held a group discussion about what had happened.

What happens next:

- The teachers were told that we would be in touch about the next research lesson;
- We distributed two lesson ideas for teachers to look at (Properties of quadrilaterals and Real-life equations) and asked them to look at them briefly together with another teacher, with a view to considering if they would use it for their next research lesson.

Issues arising:

- Some teachers told us that they found it difficult to teach a lesson designed by someone else. We told the teachers that this had always been a concern for us, and encouraged them to adapt and re-design the lesson for their own contexts;
- One teacher asked us for a lesson on early algebra and we could not find one that was already tried and tested so we designed a new lesson together with him and he talked about his experience of this;
- Many teachers said that they wanted to find ways for students to keep a record of what had happened in the research lessons. We said that we would think about how to make a recording sheets;

- We discussed that we, and most teachers, wanted to have an understanding of the students' views on the lesson, and it was agreed that we would give students a short questionnaire in the next round of lessons.

Cluster meeting 2: 18th June

This meeting took place at the main AIMS building in Muizenberg, timed to take place after school (3:30 to 5 pm). Ten teachers and six colleagues from AIMSSEC attended.

The researchers ran the meeting, first welcoming the teachers as before. She introduced Greg and said that we had asked him to talk about his experience of the FaSMEd project so far. As this case study is about Greg, we report in full on what he said.

Greg explained that he had been asked to share some of his experiences of being part of the research project. He started by saying how much he had learnt from the experience so far. He explained that the first research lesson (on exponents) he had taught was to his Gr 8 class but that it had not been a good experience. Subsequently he taught two lessons to his Gr 10 class (Time-distance graphs and Different representations) and these lessons had been much more successful. He showed some video clips from the second and third lesson and pointed out a number of things he had noticed. He spoke about, for example, the different strategies he had seen learners using in terms of how to approach a card matching activity.

He showed a clip of two learners working on their own (without teacher intervention) trying to decide how to match two pairs of similar cards. He remarked about how valuable and interesting he found this, saying how seldom teachers get the opportunity to observe the students in such depth.

When showing the clip of a group of learners putting the solutions on the board he pointed out how interesting it was that the other learners who weren't finished continued working and did not look at the answers on the board.

The last clip he showed demonstrated that as a teacher he had "nothing to do". The learners were all engaged in the task and he was moving around the class observing. He spoke about how unusual this felt but how good it was to see the learners fully engaged.

Finally he asked whether teaching a research lesson to an "easy" class was worthwhile for research and whether it wasn't perhaps more worthwhile to teach the lesson to a "difficult" class.

We showed a video montage as before and we asked the teachers to consider the following things while watching the video: the role of the teacher, the role of the students, the design of the tasks and the use of cards.

The teachers then worked in groups to share their experiences of teaching the same or similar lessons.

In terms of issues arising this term, the following were raised:

Time: we discussed the fact that the FaSMEd lessons tend to take more time than one class period and the need to take this into account;

Guidance: we talked about the extensive guidance provided with many of the lessons and the teachers seemed to think it was useful but said also liked the abbreviated lesson plan (one page) we had begun to produce.

Other points: we also talked about designing lessons from scratch, adapting other people's lessons and possibly video recording a pair of learners.

Danny Parsons, who worked with us for three months, presented some ideas related to the use of geometry software in the mathematics classroom.

Cluster meeting 3: 15th October

The third cluster meeting was held at a local school. It is a school where two of the FaSMEd teachers work.

The researchers led the meeting, beginning with a welcome and then providing a quick summary of the lessons taught that term. They then showed a video-montage, as before, asking the teachers to consider the following things while watching the video: the use of big cards, learners' response to the lesson and the use of mini whiteboards.

One of the teachers, Regis Magama, introduced the 'Revision Lesson' we had designed for his class. The teachers and AIMSSEC colleagues were then paired up and worked together on the revision lesson.

The teachers then completed a questionnaire about their experience of taking part in FaSMEd and there was then a discussion of issues arising this term:

Use of big cards for modelling and to end off. We had intended that Jane and Severino lead the discussion on these, but both were unable to make it to the meeting;

The development in the students: Berenice led a short discussion on her observations of how the students in her class had matured over the course of the three FaSMEd lessons;

Regis talked about the use of mini whiteboards in his classrooms, explaining how they helped him gather the information he needed for formative assessment.

Did you appraise the session/s with teachers?

Appraisal of sessions with the individual teacher

We did a final interview with Greg (video recorded) to ask him about the last lesson and the experience of all three lessons. This was not really an appraisal of the sessions as such but was rather an appraisal of the whole process. Greg's main points were that he and the class had become more comfortable with the style of lessons over time and that together we had all (the students, Greg and us) got better over time.

We also asked all the teachers, Greg included, to complete a questionnaire mainly about their experience of the different aspects of the research lessons, and Greg's responses to the questions relevant to this section are given below:

Post-lesson discussion and interview: 'This was useful for reflecting on how and why certain things happened in the lesson. It made me think more deeply about each of the topics.'

Reading observation notes: 'I found this less useful as they were so accurate and well done that there was nothing left to discuss.'

Watching the video of the lesson: 'Interesting and much less embarrasing than I thought it would be. It also made me aware of how I sound and react in the classroom'.

Sharing experiences with other teachers at the cluster meetings: 'I found this very interesting and useful. I picked up good ideas which I will use in future lessons.'

Watching video of other teachers' lessons at cluster meetings: 'This was ok and interesting if it was a lesson I had been involved in presenting, otherwise it became a bit boring.'

Which tools and strategies, materials, techniques, … were supportive in terms of participants' work in school?

'Active learning' approaches and pair work, especially with the use of card matching strategies, were supportive. The researchers were supportive in preparing all the materials and bringing them along to school.

What were the constraints (mentioned or experienced by teachers) for teachers' work with FASMED tools and strategies?

The main constraints observed in Greg's classroom were:

- Time, as it takes more than one class period to complete one of the FaSMEd lessons. However, Greg was aware of this and allowed two class periods;
- Teaching someone else's lesson; Greg said that he found this difficult for the first lesson and even the second one, but that it had become easier later, as he became more confident;
- Finding a 'good' class for the research lessons: as explained above, when Greg used his Grade 8 class, the experience was not comfortable for him. When he used the Grade 10 class, both he and the students reported enjoying it.

3- Classroom teaching (based on teacher interviews)

Greg qualified as a teacher of mathematics just over thirty years ago. He did a BA in mathematics and philosophy at the University of Cape Town, and then went on to do a Higher Diploma in Education (HDE) which was the name for the initial teacher education qualification at the time. He wanted to become a teacher in part because when he was in the navy (one or two years' military service was compulsory for all young men at the time) he had tutored some of his colleagues in mathematics and found he was good at it and enjoyed it. He also had some very good teachers at school and had been inspired by them.

Greg has taught at Fish Hoek High School since he began teaching. For him two main events stand out. Firstly he was made head of department in his second year of teaching. Secondly, the high school (Grades 10, 11 and 12) amalgamated with the middle school (Grades 8 and 9) in 2009. This meant that he began teaching younger students and for him this was a big change and something of a challenge.

In terms of teaching, Greg says that he likes to think that his style is not just 'chalk and talk'. He tends to use minimal instruction and then ask the students to work on examples while he circulates, looking at what they are doing and asking questions to move them on. The students in his classes often work together, usually in pairs, and they discuss the mathematics and help one another. For him, pair working is a deliberate teaching strategy, and he says he thinks it is effective because this 'gets them to do the work'. For whole class teaching, he sometimes adopts a strategy of talking quietly because, he says, the students have to be quiet to hear him.

He says that when he sees a student making a mistake, he tries not to tell them the answer, but rather aims to help them see their mistake by asking questions or providing similar or contrasting examples. When he sees students having particular difficulties, he asks them to come for an extra lesson.

He sees all areas of mathematics as important but it important particularly important for him that his students begin to learn to think mathematically; it is important to 'get their minds to work'. In terms of the difficulties students experience, he says that one main difficulty is when they arrive at senior school with 'holes' in their knowledge and understanding because they come from different feeder schools which emphasise different topic areas.

For teaching mathematics, he finds the computer and data projector very useful. He uses it to mainly to show videos, construct graphs (e.g. using geogebra) and as a graphical calculator. The students use calculators. Further experience of using technology in the classroom includes the mimeo and (previously) overhead projector.

Greg says that he has had some experience of using formative assessment. For him, formative assessment involves students learning with other students, without a textbook and appealing to different learning styles and senses. As an example, he described a lesson in which students did 'co-operative' learning on exponents. Each small group worked on one aspect of the topic and then fed back to the bigger group. More recently he has used the formative assessment lessons suggested by the Fasmed team. He says, however, that these sorts of approaches are better suited to some topics than others and that one barrier to using them more often is the time need to prepare.

Greg said: 'I think that one of the advantages of formative assessment is that it is less threatening. Kids are more likely to try as there is not quite the same pressure if they are "wrong". This is especially true if it is one to one with the teacher (so not in public) or within a peer group.

Another advantage is that it allows more capable students to progress at their own, quicker pace. At the same time the slower ones do not really get left behind. One can have material available for those who get ahead. This would/could be of a more challenging nature.

Your example of them seeing the mistakes of their peers and working collaboratively was also clear both in your lessons and in class at other times.'

4- Lessons

Organisation of the classroom observations

All four of Greg's research lessons were video recorded, with two researchers in the classroom. Usually one researcher video recorded and the other took notes, and some of the time one or both circulated and observed the students working.

An a priori analysis of the lesson(s)

Greg taught four FaSMEd lessons. For all FaSMEd lessons, we decided to use lessons that:

- began with a brief whole class introduction in which the teacher assessed the students' knowledge and understanding of the topic to a greater or lesser extent and told the students what they should do in the lesson;
- adopted so-called 'active learning' approaches, with student working in pairs or small groups, mostly to sort or match sets of small cards. This provides the students with the opportunities to discuss and learn from each other, formatively assessing one another to guide some of their choices;
- expected each group to produce a poster with the card matches stuck down on the poster, and these posters might be shared with other groups;
- included a phase of institutionalisation such as sharing posters or going through the answers together.

To some extent, therefore, the didactical choices were already made for Greg.

The more subtle choices, such as how and when to intervene in small-group discussions or activity, tend to be made by the teacher and are determined by the individual's preferences and their reaction to the data they gather in real time about their students' understanding.

Cards were our main technology; we did not use ICT, apart from display technologies in some classrooms (Greg's was one of them) but ICT had no real role in the task design. On the other hand, small card sets were central to the design. The use of small cards by students can provide the teacher with a window into the students' thinking.

In a sense Greg chose the lessons he wanted to teach by telling us which topic he would like the lessons to address. For three of the lessons, the topic was one his class was addressing at the time so the lesson fitted into the scheme of work. For the fourth, which was the last lesson, Greg had asked for a lesson on a topic that he knew his students found difficult and he wanted them to revisit the topic before the end-of-year examinations. These lessons were all designed as ‘one-off’ lessons to be taught after the students have already been working on the topic for a little while.

Although the four lessons followed a similar structure, the actual mathematical task the students were asked to do was different in each case. We briefly analyse the tasks below. It is also perhaps useful to consider, *a priori*, the obstacles inherent in each task as identified by Brousseau (1997): ontogenic obstacles which relate to the readiness of students for the task; didactical obstacles which are teaching and classroom obstacles; and epistemological obstacles, which require the students to adapt their strategies to solve a problem. The first two types of obstacles should be avoided, but the third should be present.

Properties of exponents

This is a card matching activity, with two sets of cards: one set of single exponents and one set of expressions. It is designed to challenge students by presenting them with expressions that can be simplified by applying the ‘laws’ of exponents (e.g. $6^8 \div 6^4$), and others for which no law applies $2^2 + 2^3$. The students need to recognise when a law applies, and then to apply the appropriate law accurately. When no law applies they need to find another way to evaluate the expression and for many students this presents challenge which can be seen as an epistemological obstacle. The cards fall into ten groups and the matching is not one to one. Some blank cards are provided in case students want to make up their own cards.

This task was used with a Grade 8 class, who had recently worked on the rules, or laws, of exponents but were perhaps not ready for the complexity of the task, particularly in terms of recognizing when a law could not be applied. It is possible that this presented an ontogenic obstacle. In terms of the classroom teaching, they also had little or no experience of working in groups on card-matching activities, which could also develop into a ontogenic obstacle. *A priori*, therefore, it was possible that the students would not perform as well as hoped.

Time-distance graphs

This is a card matching activity, with two sets of cards: one set has descriptions of Tom’s activity and the other has graphs corresponding to the description. The students match the graphs with the corresponding description. The cards are carefully designed to reveal students’ misconceptions and then for students to realise that they have made mistakes because they cannot match all the cards. The cards are designed so that common misconceptions are revealed; particularly the tendency of learners to confuse the shape of a graph with a picture. For example, suppose they are given a description such as 'Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.' Many students would match this with a graph that looks like a 'picture' of a hill. Other graphs differ from others in subtle but important ways and for students to work out which graph matches with which description requires careful reading of the graph. These challenges would represent epistemological obstacles for the students.

The task was used with a Grade 10 class, who were familiar with time-distance graphs and had some experience of interpreting graphs. They had some enthusiasm for mathematics and appeared willing to try new things, so it could be that ontogenic obstacles were avoided.

Multiple representations of algebraic relationships

In this activity students work in pairs to match various representations of functions: the equation, the graph, a table of x- and y-values and the 'rule' in words. The cards include the following relations (not all functions): a straight line graph with a positive gradient, a vertical line, a horizontal line, a 'happy' parabola, an 'unhappy' parabola, two exponential functions (one with a shift), two hyperbolas, sine, circle and a square root (inverse quadratic). One graph card is left blank and the students need to draw the appropriate graph. Similarly one equation card and one rule card are left blank and some cells in the tables of values are left blank. This task is designed to emphasise the multiple ways a function (or non-function) can be represented. In working through the task students will need to use, and pull together, their prior knowledge of graph shapes.

The challenge for the students is partly in sifting through the information and finding a way to organise it (epistemological obstacle).

Function notation: when is f(x) > g(x)?

In this task, two sets of cards need to be matched. The first set of cards are graphs of two functions, f(x) and g(x). One graph page is left blank. The second set of cards have statements that are true of at least one of the graphs. There is one for which there is no matching graph, however. Learners need to match the statement cards to the correct graph cards. Where a statement card matches more than one graph card, they should make another statement card. A key idea behind the activity is that it is not necessary to know the equation of any of the graphs; the information can be read from the graph. In terms of epistemological obstacles, this idea is key. So also is the need to distinguish between, for example, a function taking a negative value and a decreasing function.

The task was used with the Grade 10 class who had already used two other FaSMEd tasks. In terms of the ways of working, it seemed unlikely that didactical obstacles would get in the way of learning. However, some ontogenic obstacles may have presented themselves; the lesson was taught some time after function notation and the students may not have been prepared in terms of their prior knowledge.

Organisation of the lessons

Research lesson 1: (Grade 8 exponents)

Lesson 1

Activity 1: The students were given the questions shown in Figure 5 (see above) to answer individually. These are questions designed to reveal the students' current levels of understanding and can be used by the teacher to inform what he does next. The teacher marked the work and showed it to us. He said he was disappointed but not surprised that the students did not do very well.

The teacher's analysis of the assessment is shown below in Figure 27.

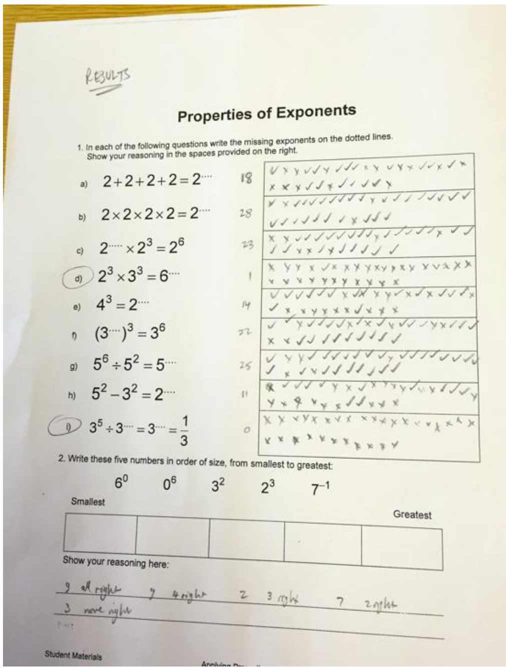


Figure 27: Teacher's analysis

Lesson 2

Activity 2: Brief discussion of the pre-lesson assessment task.

Activity 3: The teacher displayed the PowerPoint slide 'Powers of Two' (Figure 6) and asked the students to write first the statement A (8 x 4 = 32) in powers of two on their mini whiteboards and then to write the other three statements (B, C and D) also in powers of 2. As they worked, he circulated, discussing the students' answers with them, and asking some groups to compare their answers with other groups.

Activity 4: The students were asked to work in groups of four to put the cards shown in Figure 7 into groups with equal value. They began working on the cards and Greg circulated, stopping to talk to the groups from time to time, as shown in Figure 28, below. He said later that he noticed that very few of the groups seemed able to cope with the cards which required students to work out values (such as $3^2 - 2^3$) and that he decided to ensure that this was explicitly addressed in the final whole class discussion.

Figure 28: Discussion with a small group

Activity 5: Students shared work, see instructions in Figure 10. The students seemed a little confused, as not all groups were visited. However, some visiting did take place. Figure 29 shows a girl visiting another group. Her discussion with them lasted about three minutes.

Figure 27: Sharing work

Activity 6: Whole class discussion: going through the answers. Greg began by discussing the cards: $2^3 \times 2^3$; $(2^3)^2$; 2^6; and 4^3. He said that they were all four the same and asked if any group had got all four. One group put their hands up. He then went through $3^2 - 2^3$, saying that for this one they had to calculate the values and asking what the value was. He then discussed other examples, and spent some time on $2^3 \div 2^{-2}$, reminding them that the exponents should be subtracted: 'that's what the rule says'. He wrote $2^{3-(-2)}$ and asked the students what this gives. He said they should remember subtracting negative numbers from earlier in the year. One student volunteered 2^5 and Greg agreed that this was the correct match.

PRE-ASSESSMENT				POST-ASSESSMENT				DIFF
18	6	24	100	18	6	24	100	0
8	2	10	42	4	0	4	17	-25
14	5	19	79	12	4	16	67	-12
13	4	17	71	12	0	12	50	-21
16	5	21	88	15	5	20	83	-5
6	4	10	42	10	4	14	58	16
6	4	10	42	12	4	16	67	25
14	5	19	79	16	5	21	88	9
8	0	8	33	6	1	7	29	-4
3	2	5	21	6	4	10	42	21
10	2	12	50	12	4	16	67	17
8	4	12	50	6	6	12	50	0
8	4	12	50	7	2	9	38	-12
14	5	19	79	12	4	16	67	-12
7	4	11	46	4	4	8	33	-13
10	4	14	58	8	4	12	50	-8
7	4	11	46	10	4	14	58	12
10	4	14	58	6	3	9	38	-20
7	4	11	46	12	3	15	63	17
10	4	14	58	8	5	13	54	-4
11	2	13	54	12	4	16	67	13
12	3	15	63	12	4	16	67	4
7	2	9	38	8	4	12	50	12
6	4	10	42	10	3	13	54	12
8	4	12	50	8	4	12	50	0
8	3	11	46	12	4	16	67	21
0	0	0	0	3	0	3	13	13
6	2	8	33	8	5	13	54	21
5	0	5	21	6	4	10	42	21
6	2	8	33	4	0	4	17	-16
7	4	11	46	10	4	14	58	12

Figure 30: Students' grades

Lesson 3

Activity 7: Students completed a post-lesson assessment, as shown in Figure 11. Greg compiled a spreadsheet to compare the pre- and post-lesson assessments (Figure 30) and told us that he was disappointed. As can be seen, some students performed better on the pre-lesson assessment (e.g. #1) whereas others did better on the second assessment (e.g. #5). He said he thought the class had not benefitted as much as he hoped and he thought it was because they did not take the work seriously.

Research lesson 2 (Grade 10, time-distance graphs)

After Research Lesson 1, Greg switched to using his Grade 10 class.

Lesson 1

Activity 1: Pre-lesson assessment, see Figure 12. Greg said he did not mark it but he read through the students' responses and thought from these that they had a reasonable understanding of the topic.

Lesson 2

Activity 2: Class discussion: choose the story to match the graph (Figure 13). Greg asked how many had chosen Story A. We saw that one or two hands were raised tentatively, but were quickly taken down. He asked how many had chosen Story B and two or three hands were raised. He asked why and then discussed that the horizontal axis was 'home', so the graph represented Tom leaving home at the beginning and coming back home at the end. He said that Story B could not match the graph because in Story B Tom gets further away from home. He then asked who thought that Story C matched the graph, and most (or all, possibly) students put up their hands. He selected one student, Angela, to explain. He then analysed the graph in detail, asking questions such as 'What does this point mean?' [furthest point from home], 'How do you know if he is going fast or slow?' [steepness], and 'How do I know he is running away from home or towards home?'. He said that he supposed he would have to tell them, and said 'The point I am trying to make is that if the graph is going up to the right, then which way is he going? Think about it.'

Activity 3: The students were asked to work in groups of four to match the cards shown in Figures 14 and 15. Greg said to the class that someone in each group should record the matches. He pointed out that the cards had a little number or letter and they should use these to record the matches. He then circulated, briefly checking that the students were getting started and reminding each group that someone should record the matches.

Greg went to look at a group of girls at the front of the classroom (Figure 31). They continued with their discussion for 30 seconds while he looked at the cards they had already matched, then pointed at one match and said 'Explain this one to me. It says that he walks up the hill and then runs down the other side. Does he run down the other side back to his house? Does he get back home?'. The girls seemed to think their match was correct (I couldn't see but I am guessing it was Card E or Card G, which look like a mountain) and Greg shook his head. One girl laughed and said that they had made a silly mistake and Greg walked away, also laughing. The girl explained the mistake to the others in the group.

Figure 31: Talking to a group of girls

At another group of girls he asked them if they would be able to explain their matches if someone asked them or disagreed with them. They nodded their heads, and he said 'Good stuff' before moving on to the group of boys at the back.

Activity 4: Students shared work, see instructions in Figure 16. Greg read through the instructions with the class and then explained again that one person from each group should visit another group with their matches and compare.

Activity 5: Institutionalisation, reading out the answers. Greg said to the class that it seemed that there was general agreement. He asked if there were any 'big arguments'. He said they were going to do one more thing but first they would go through the solutions and check their matches. He read the answers from the lesson guide. At the end some students said 'Yay!' and Greg asked 'Is everyone happy?'.

Greg asked one group to read out their own story (Card 10), which involved someone leaving home and going to point A, going back part of the way and then going back to point A (I couldn't hear the details). Greg said that he assumed everyone had the same story but with different contexts.

Activity 6: Students worked individually to make up data for a graph, see Figure 17. Greg showed the slide 'Making Up Data For a Graph' and said that in the packs on the tables, they would find a printout of one of these for each student. He said they should quickly fill in their own distances. One student commented that they would all get different distances, and he said that they would. They worked in near silence, individually.

Figure 32 shows the work of one student, whom I observed. She used a ruler to measure the height of the graph (i.e. the length of the dotted line from the time axis to the graph, in mm) and filled these lengths into the table.

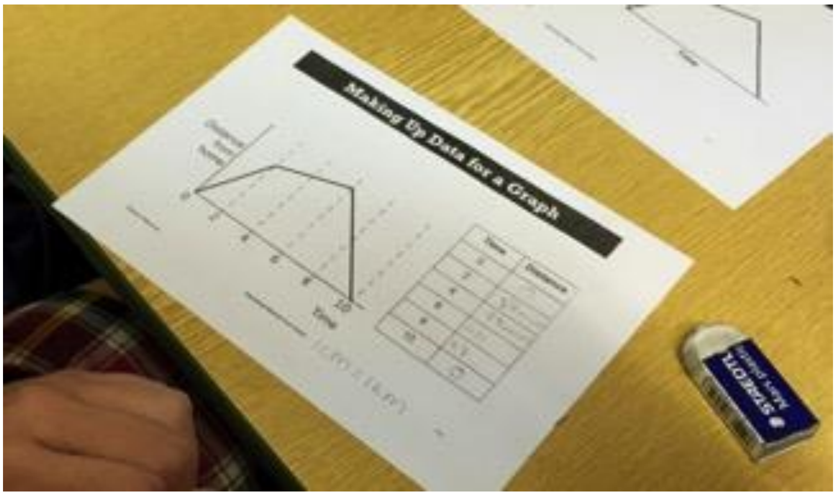


Figure 32: Making up data for a graph

Activity 7: Card matching in pairs. Greg handed out set of yellow tables cards, and said that now that they had an idea of how the distances are related to the time, could they match the yellow table to the graph. They began to do this and Greg circulated, visiting each group for a few seconds. Figure 33 shows one group's early matches.

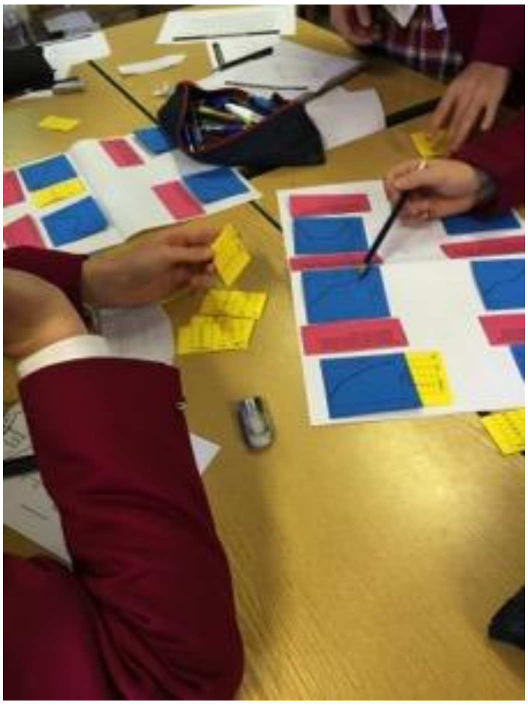


Figure 33: Matching the table cards

Lesson 3 (not observed)

Activity 8: Students completed the card matching. Greg told us afterwards that the lesson the following day had not taken very long. He said that most groups agreed on the matches; one group had an incorrect match but as soon as it was pointed out to them, they understood their mistake and corrected it.

Activity 9: Students completed a questionnaire about the lesson. The results are reported in Section 5 – Pupil perceptions.

Activity 10: Students completed the post-lesson assessment. Greg sent the results to us. They suggest that there was some improvement in the students' understanding. Greg said that he thought it was 'a productive lesson in which all seemed engaged.' He added that the students felt loyal to their groups in the next lesson.

Figure 34, below, shows a classroom display with a completed poster, quotes from the student questionnaires and photographs from the lesson.

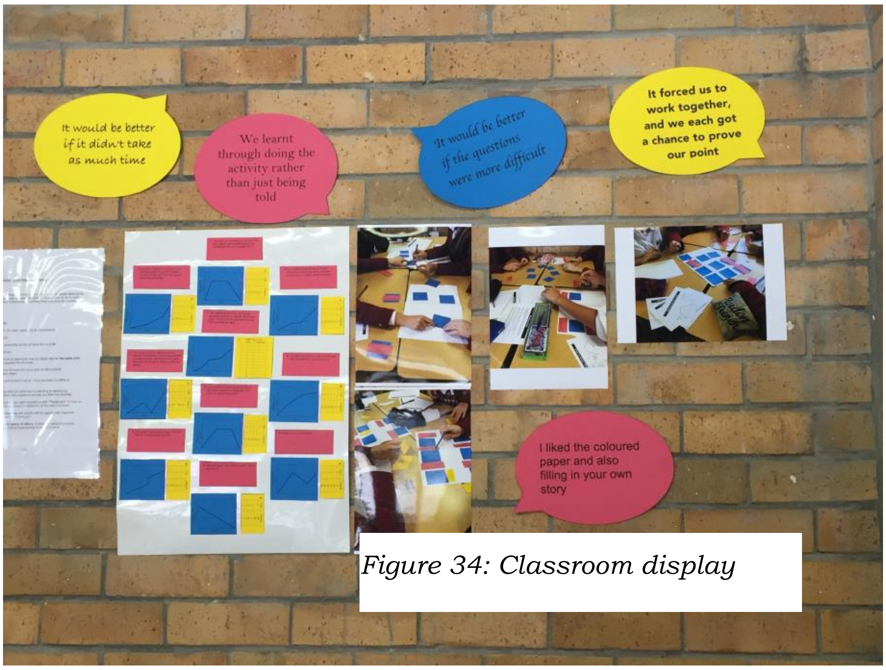


Figure 34: Classroom display

Research lesson 3 (Grade 10, Multiple representations of algebraic relationships)

Lesson 1

Activity 1: The teacher modelled the activity using big (A4) cards made for the purpose (see Figure 20, above). Greg used the big card of the straight line graph we had prepared for the modelling of the activity. He asked how the same information could be represented in another way. Some one suggested 'points' and another said a table and Greg agreed. He put up the table card, which has some values missing, and said 'it wouldn't be too difficult to complete this table, would it?' He explained, demonstrating on the graph, that we can find the x-value and read off the corresponding y-value. He then asked what other way could be used and someone volunteered 'formula', and Greg said yes, an equation. He found the equation card, but before sticking it up on the board, asked what kind of equation they would expect and they had a brief discussion about the form of the equation ($y = mx + c$ or $y = ax + q$), the y-intercept and the gradient. After sticking up the equation card he asked how else it could be represented and someone suggested 'in English' and they talked through how it could be said in English. He wrote this on the board, as shown in Figure 19. Later he stuck the green card up explaining that he didn't want to write on the green paper (which is blank).

He asked if everyone was 100% happy with what they had just done, and then explained what he wanted them to do.

Activity 2: Students were given four sets of cards: equations, graphs, tables of values and descriptions in words (Figure 22). They were asked to put them into groups. Greg suggested to the class that they should stick the yellow graph cards 'at the top' and then group the other cards below them. He used Prestik to stick all the yellow graph cards to the whiteboard. He left the other big cards at the front of the classroom, telling us that he wanted to see what happened. He said that he hoped that the learners would spontaneously come up to the board and begin placing the other cards under the graphs in their groups.

Whereas some groups followed Greg's instruction of starting with the yellow graph cards (Figure 35), others grouped the cards in different places around their poster (Figure 36). However, it seems that for most groups, the graph was used as the starting point.

Figure 28: Starting with the graph cards at the top

Figure 29: Grouping the cards anywhere

Most groups appeared to match the blue equation cards to the graphs as they began the activity, mostly then going on to match the green rule cards and finally adding the pink tables to the groups. This order of card matching can be seen in both Figures 35 and 36. Some others began with the pink cards, and Greg remarked later that it seemed that these groups were slower to finish.

Figure 37, below, shows the in-progress work of one group who adopted the approach of groups the cards they thought were easier at the beginning. When Greg visited this group, he suggested that 'part of your problem is that if you do this [pointing at the groups of cards in little piles] then you can't see what you've got'. He asked them why they had made the little piles and they explained 'those are the ones we *know*.'

Figure 30: Grouping the cards 'we know'

They asked him about the table T6, shown in Figure 38, and he asked them which graph had a -8. They pointed to a graph and said they thought the table did not belong to this one and Greg suggested checking that none of the graphs in the little piles had a -8 value for y. One of the students responded that there was no point as these were the ones they knew. The other remarked that they should look through the little piles as she thought she wasn't sure that she knew anything. Greg asked some further questions, mainly about other cards, and left them.

T6

x	-2	-1	0	1	
y					-8

Figure 31: Card with table T6

Greg told us he would continue with the activity the next day. He asked the learners to listen carefully. He gave some instructions about pasting down the cards with Prestik, and safely putting aside the ones they had not yet decided on, and putting their names on the posters.

Lesson 2

Activity 3: Students continued with the card matching activity. The lesson began at 11:40. The students came into the classroom, and Greg asked them to find their posters and continue with the previous day's activity. Greg circulated, checking the students' work and asking questions.

For example, one group had placed the equation $y = 2^{-x} + 2$ with the 'unhappy' parabola, and had also correctly placed $y = 2^x$ with the exponential graph on card G8. He pointed out that both equations had x as exponents, implying that they might match with similar graphs, but saying that the exponential graph and the parabola didn't look similar to him. He said that one of them must be wrong and asked them to think a bit more.

Activity 4: Two groups of students, who had completed the activity, were asked to go to the board, and to select a set of big cards and paste them on the board. See Figure 39 for a photograph.

Figure 32: Pasting big cards on the board

They did this and then did the other groups as well. They also filled in the blank cards and gaps in the tables. Throughout this they referred to their own posters.

Greg said afterwards that he had intended that they should only do one group but when he saw them doing the others, he decided to let them continue. He also said that he had thought that he would ask them to explain their matches, group by group, but decided against this as in some cases it hardly seemed necessary, it would take too much time and it now no longer made sense as the groups were all getting stuck on the board.

As these students were sticking the cards on the board, others were still working on their own posters. Very few of the others looked at the activity at the board and they seemed intent on getting their own posters completed.

Activity 4: Whole class discussion. The teacher went through the card matches. Greg called the class to attention. He said he was going to start by filling in the blanks (only the graph G1, the 'happy' parabola, needed to be filled in). He began by plotting points and drew the graph.

He briefly discussed the group of cards going with G1 (E3, T10, R7).

He then discussed the group matching G2, the straight line graph, stating that this was straightforward. He said they would skip G3, as they were going to come back to it. For the group of cards for G4 (hyperbola) he asked 'what was the give away?'. The learners responded 'infinity'.

He then drew their attention to G5 and G9, the vertical and horizontal straight lines, and asked if everyone agreed with the matching cards, and it seemed they did. He asked 'does anyone disagree with anything so far?' and no-one said anything.

He went on to the second hyperbola, asking what had made them think the cards would match? One of the students responded with 'the minus 2'.

He then pointed at the 'unhappy' parabola, stating 'there was no unhappy graph equation so we had to make it up. How do you know what it is?' Some people said you need to look at points on the graph and someone else pointed out that the green tells you, suggesting what the equation should be.

For G11, the sine curve, Greg said this was 'very easy'.

Finally, Greg said 'now we come to the ones we haven't seen before, G3 and G12'. He said 'G3, talk to me about it, how did you match it up?' One student responded by saying that they had matched the points on the table to the graph. One girl said 'it was easy, it was the only one with 16.' Greg asked 'but do you understand?' and the same girl said she did not. Another student suggested that it was like a parabola and there was some discussion about how the equation could be re-written with x as the subject.

For the circle graph, Greg asked 'it didn't phase you?' adding 'I want to know did you understand it?' One boy said 'we found the 2 and the 2 was the only one that had those points.' Greg pointed to the equation, asking 'what does look like?' and someone responded 'Oh, Pythag.' Greg pointed out that 22 is r2.

To finish, Greg held a short discussion about functions. He asked 'what makes a function?', reminding them that for a function, 'whatever the input it you get a unique x value.' He asked them to look at all the graphs and see which are functions and which are not. He asked 'how can you tell very quickly which of these are functions?' A girl volunteered a response, going to the board and pointing out that G3 has two x values for each y value (Figure 40), and Greg said that if a vertical line is drawn and cuts the graph in more than one place then the graph does not represent a function, adding that this is called the straight line test. He asked is anyone had any questions but it seemed they did not. Soon after this the lesson ended.

Figure 33: Demonstrating two y-values for one x-value

Lesson 3 (not observed)

Activity 5: Individual work. The students were asked to fill in the recording sheet and stick it into their books.

Activity 6: Individual work. The students completed a questionnaire. The results of this are discussed below in the Section 5 – Pupil perceptions.

Research lesson 4 (Reading functions)

Lesson 1

Activity 1: Brief introduction; the researchers explained to the students what they should do. Marie explained (Figure 41) that each pair of students would be given a pink set of cards (graphs) and a green set (statements) to match and that:

Figure 41: Explaining what to do

1. each pink card should have a matching green card
2. some green cards could go with more than one pink card.
3. there was a blank graph card (yellow) to be used if there was a green statement card without a matching pink graph card.

Activity 2: The students worked in pairs to match statement cards with graph cards. Each student was given a mini-whiteboard for rough working or making notes.

Greg and Marie circulated and observed what the learners were doing. Most got started quickly and began making their matches, see Figure 42.

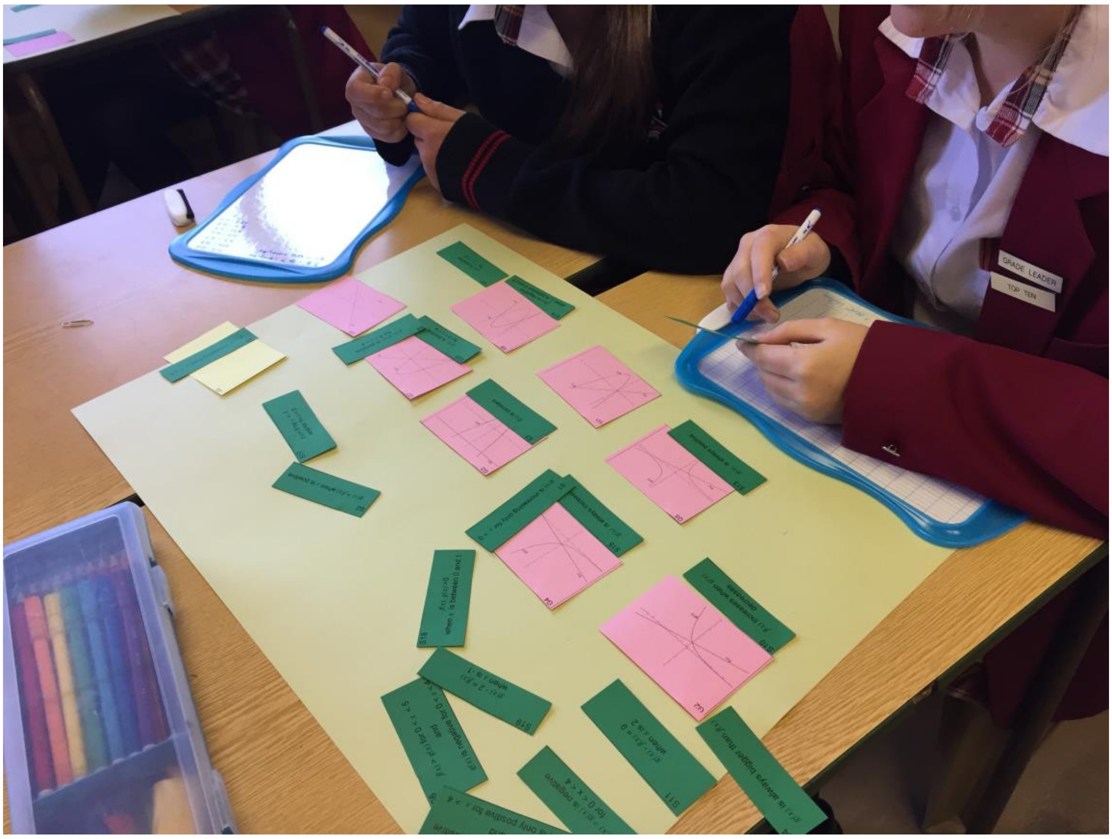


Figure 34: Beginning to match cards

Greg mainly asked questions or gave hints rather than telling learners what to do. For example he pointed out to one group who were unsure about two cards that: "It's right for part of it but not always." To a number of groups he said '*f(x)* is negative…that means it's below the *x* axis' or '*g(x)* is positive…that means it is above the line.' He asked questions like 'If you multiply two things and the answer is positive them you know what?' A pair of boys asked about card S19 (*g(x)* - 2 = *f(x)* when *x* is -1) and Greg asked them 'How are you understanding what that is saying? What if you converted it into English?' When they answered he said 'You've got the idea, now go and find it!'

A few groups told Greg that they felt overwhelmed or that they didn't feel comfortable with graphs and Greg told them that this activity would help them improve. With these groups he generally did an example to 'help them get started'. After one such demonstration he said to the group: 'Can you see how I'm thinking? Now you start thinking like that.'

One pair of girls couldn't find a match for S4 (*g(x)* is always bigger than *f(x)).* Greg reminded them that they had blank graph card which they could use if there was no existing graph card that could be matched.

One pair of girls appeared very pleased that they were allowed to draw ANY two graphs on their blank graph card (as long as the one was always bigger than the other). They said 'let's see how many different ones we can come up with'. Figure 43 shows three graph solutions suggested by different groups of students.

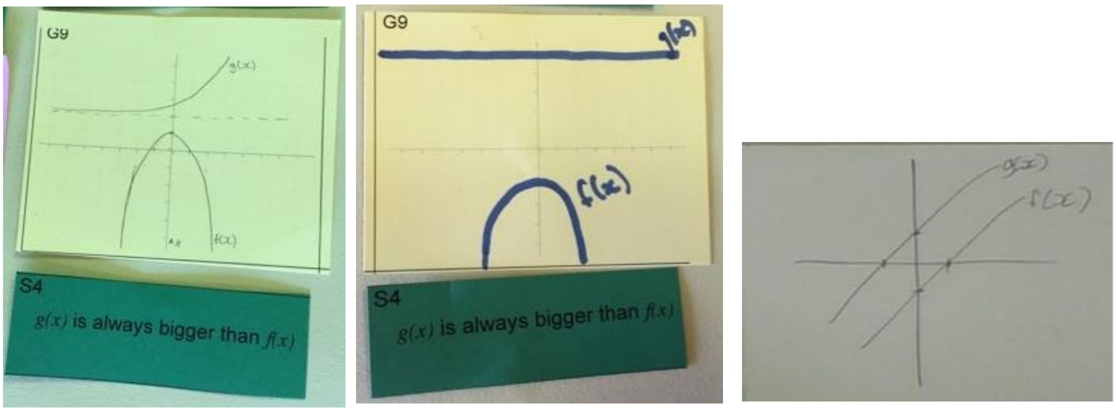


Figure 43: Graphs made up by the students

The same group of girls was unsure how to stick down the statement cards if the same card applied to more than one graph card. This was something that we had not finalised in our design. Greg told them that they could choose which graph to stick the statement card under. The girls appeared not to be satisfied with this solution and so although they did stick the card down under one card they also used their mini whiteboards to record all the graph cards that it could be matched with.

Greg stopped the class, and said that they would continue the following day and took in the mini whiteboards and the posters. He told the class to write their matches on their mini whiteboards with their names and to stick their cards on their posters and also write their names on the posters.

Lesson 2

Activity 3: Students continued to match cards. About a third of the learners were absent as they were on a geography trip so Greg had rearranged the desks so that 4 desks were together to allow for groups of 3 with enough space for two or three posters (see Figure 44).

Figure 44: Desks arranged to accommodate multiple posters

It was the end of break and the learners started arriving. They found their posters and started working. Where there was more than one poster on a desk, they started comparing posters.

Activity 4: The teacher pasted all the big graph cards along the top of the board. He handed out two or three big statement cards to each group and asked them to paste them on the board in under the appropriate graph card. One group asked what to do if their green card matched more than one pink card. Marie said they could use other copies of the same card (from the spare small ones) and therefore stick a copy of their card under all the graphs it applied to. Figure 45 shows all the cards stuck on the board including little cards to indicate statements applicable to more than one graph. One of the learners asked whether they could move a card if they saw that it was wrong. Marie suggested that the class should decide. One of the girls said that the cards should only be moved during the class discussion so that the people who had placed it could defend their decision.

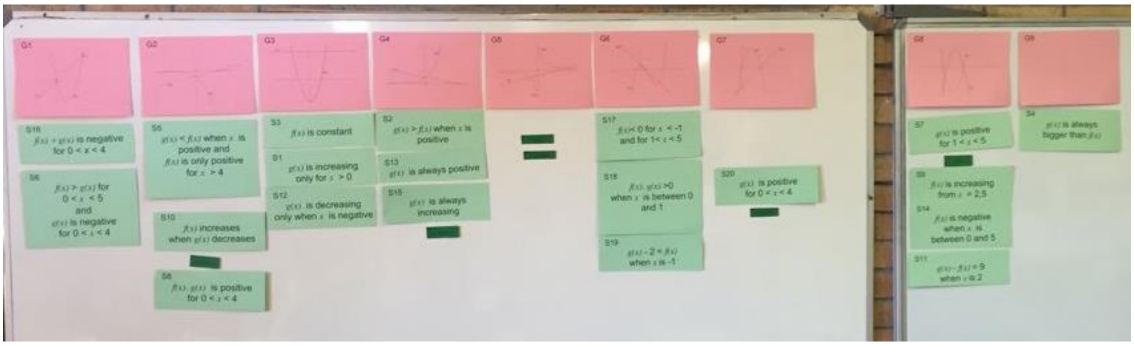


Figure 45: All cards stuck on the board

Activity 5: Whole class discussion, the phase of institutionalisation as the teacher went through the card matches. Greg stopped the class and said that they were now going to discuss the cards. He encouraged them not to be scared to say what they wanted to say. He started going through the cards. He looked at each of the graph cards, one at a time, and checked each of the statement cards that had been stuck under it.

Some cards generated a discussion while for others Greg only confirmed that the cards were correct. Where cards had been placed incorrectly learners explained to the class why the matching was incorrect and the statement cards were then moved to the correct graph. Learners also mentioned when they had a statement which could be matched with a graph but had not been. In this case an extra card was stuck up.

Activity 6: Whole class discussion led by the researchers about the design of the lesson. Marie asked the students some questions, and the discussion is reported in Section 5 – Pupil perceptions.

Analysis of the Research Lessons

Introductory comments

The lessons used by Greg have already been analysed briefly above. The important point to make, at this stage, is that the design of the lessons lends itself to active learning. Further, the lessons are designed as formative assessment lessons, and although teachers and students may not use the information they gather in formative ways, it is likely that some formative assessment will take place.

This section provides an analysis of the lessons in terms of the teacher's approaches, the student engagement, the use of the technology and formative assessment.

The teacher's approaches

In this section we address questions such as:

- Did the teacher engage the students? Did the teacher promote student learning?
- How did the teacher give feedback to the students? (was the feedback inspiring for further inquiry or did the feedback limit further inquiry by providing solutions).
- How did the teacher motivate the students to become active learners in his/her classroom? What motivation tactics did the teacher employ to keep students focused on the subject?
- What type of questions did the teacher pose? And how did he pose them (orally, using technology, to the whole class, individually…)?

The teacher's role

The teacher's role before the classroom intervention was to work with the researchers to plan the lesson, deciding which parts of the proposed lesson to use and how. For example, he might decide to use a pre-lesson assessment or not. He might decide to cut short the whole class introduction to the main activity, to extend it or to adapt it in the light of the students' responses both before and during the lesson.

In the main lesson, the teacher's role was to introduce the main class activity, to supervise (in some way) the group work, and to bring the lesson to a close.

Pre-lesson assessment

For two of the lessons (first and second) the teacher gave a pre-lesson assessment task to the students, which aimed to find out something about the current levels of student understanding. In the first of these, Properties of Exponents, the assessment revealed that students found many of the questions difficult but they seemed confident about applying the rules they already knew. Greg said that he used this information in the main lesson because it suggested to him what he should emphasise. In the second, Time-distance graphs, the assessment showed that the students already seemed to have a good knowledge of the topic. Greg said that, knowing this, he decided to move quite quickly into the main activity in the lesson.

Introducing the activity

In introducing the activity, in all but the last lesson, Greg stood at the board and asked the class questions. We noticed that mostly he would ask a question and one or more of the students would offer an answer. Sometimes, but not always, they put up their hands and he asked those with hands up. For example, in the Properties of Exponents lesson, he asked 'What is a power of 2?' and one student volunteered 'two times two', which Greg wrote on the board. He asked how could this be written another way. One student had her hand up and he asked her, and she said it was 'two to the power of two'. He asked for another example of a power of two and one student called out 'two to the power of five' and he wrote this on the board. He said 'this can be written in three different ways' and pointed at 2^5 on the board saying 'can can write it like this, or…?'. A student said '2 x 2 x 2 x 2 x 2' and Greg agreed and wrote this on the board, saying 'yes, 2 x 2 x 2 x 2 x 2, or… we can write it as what it actually is, which is…?' … 'what is it actually, if you did this calculation?'. One student said 32, and he said 'Ja, 32, that's right'.

During this discussion, Greg was checking that the students understood the terminology. He then asked them to write four statements as powers of two (see Figure 6 above), using mini whiteboards. There was some discussion between them as they worked and Greg walked around. One student asked what to do and Greg asked him again 'What is eight as a power of two?' and the student responded but I could not hear what he said. Greg asked more questions and then reminded the student of what he should do. He continued to circulate, and now and then helped a student, if he saw that they were struggling. He said to one girl 'Now you must write 32 as a power of two. Remember we did that on the board'. Another girl asked how to do C: 8 ÷ 16 = ½. He said 'Ok so you have two to the power of three divided by two to the power of four, so what do you get?' and she responded 'two to the minus one'. He pointed to the ½ and said 'So what is half as a power of two?'. She said 'oh! I see'. He moved to another group, and saw that they had 'only done the right hand side' and explained that they needed to write all twelve numbers as powers of two. He stopped the class and explained that they needed twelve numbers written as powers of two.

In this time, as students were working on their whiteboards, Greg was looking at what the students were writing on their whiteboards and intervening as necessary. He was gathering information about the students' current levels of understanding about the mathematics and about what they should do, and acting accordingly, by suggesting what they should do next or questioning them to move their thinking on.

For the second lesson, time-distance graphs, Greg led the introduction to the activity from the front of the room. After asking the students to choose which story matched the graph, he asked how many had chosen Story A. We saw that one or two hands were raised tentatively, but were quickly taken down. He asked how many had chosen Story B and two or three hands were raised. He asked why and then discussed that the horizontal axis was 'home', so the graph represented Tom leaving home at the beginning and coming back home at the end. He said that Story B could not match the graph because in Story B Tom gets further away from home. He then asked who thought that Story C matched the graph, and most (or all, possibly) students put up their hands. He selected one student, Angela, to explain. He then analysed the graph in detail, asking questions such as 'What does this point mean?' [furthest point from home], 'How do you know if he is going fast or slow?' [steepness], and 'How do I know he is running away from home or towards home?'. He said that he supposed he would have to tell them, and said 'The point I am trying to make is that if the graph is going up to the right, then which way is he going? Think about it.' Afterwards he explained that he had intended to annotate the graph but because the students were answering his questions without hesitation, he thought they understood well enough, so he decided not to. This is a clear example of real-time formative assessment, with the teacher gathering information and adjusting his plan accordingly.

For this last question he seemed to want the students to answer that a positive gradient represented movement away from home and negative gradient represented movement towards from home, but the students did not provide the answer he wanted. When we asked him about this afterwards, he agreed that he had been looking for the connection between positive and negative gradients and movement away from or towards home.

In the third lesson, we gave Greg big cards to model the activity. He did this in a few minutes, asking questions from the class and getting responses, usually called out. In doing this, he was checking on a general level that the class understood at least something of the mathematics, and he was showing them how to do the card matching. He finished off by asking if everyone was 100% happy with what they had just done, and then explained what he wanted them to do.

In the fourth lesson, we introduced the activity.

Greg told us afterwards that over the course of the four lessons, he became much more confident about using the card matching and his Grade 10 class (used for the last three lessons) also became more confident. He also said he thought the students knew enough of the mathematics required for the card matching activity not to need much of an introduction. He discussed in particular the introduction to the third lesson, which was short and only modelled the activity, and said that 'leaving it open like this allows them to do, to struggle, but I think that's much more valuable for them'.

Main class activity

In all four lessons, the students were given sets of cards to match. In all of them, the students seemed to be fully engaged by the activity: Greg did not have any work to do in engaging them.

Greg tended to circulate and as he did so, he could see what the groups of students were doing. He also listened to their discussions. Much of the time he did not intervene and he remarked later that he had 'nothing to do'. He talked about this in the 'Cameo slot' at the second cluster meeting, saying that this was unusual for him. He added that this gave him the opportunity to see what his students seemed to understand. Card matching activities are designed exactly to provide this sort of opportunity for the teacher, and it seems that Greg had a good understanding of how to use it.

When he did intervene, it was usually to question a decision the students had made. For example, in the Time-distance graphs lesson, as he was visiting one group, he looked at the cards they had already matched, then pointed at one match and said 'Explain this one to me. It says that he walks up the hill and then runs down the other side. Does he run down the other side back to his house? Does he get back home?'. The girls seemed to think their match was correct (I couldn't see but I am guessing it was Card E or Card G, which look like a mountain) and Greg shook his head. One girl laughed and said that they had made a silly mistake and Greg walked away, also laughing. The girl explained the mistake to the others in the group. This demonstrates Greg's preferred approach, he told us later, to ask questions to reveal errors rather than to tell students they are wrong.

Sometimes students asked Greg for help. For example, in the third lesson (Multiple representations of algebraic relationships), one group asked him about the table T6, shown in Figure 46, saying they did not know which graph it matched with, and he asked them which graph had a -8. They pointed to a graph and said they thought the table did not belong to this one and Greg suggested checking that none of the graphs in the little piles they had already put aside had a -8 value for y. In another lesson, the Reading functions one, a pair of boys asked about card S19 ($g(x) - 2 = f(x)$ when x is -1) and Greg asked them 'How are you understanding what that is saying? What if you converted it into English?' When they answered he said 'You've got the idea, now go and find it!'

T6

x	-2	-1	0	1	
y					-8

Figure 35: Table T6

Finishing off

In the final part of the lessons, Greg went though the answers. In the first lesson, he discussed the answers and wrote the groups on the board, asking the class as he went along, for contributions and for confirmation that they were understanding. When he got to the fourth pairing, $2^2 \div 2^3$, he remarked that this was the question many students had got wrong in the pre-lesson assessment. He asked what the answer should be and one student volunteered 'two to the minus one'. He wrote this down and asked if other people had got his. Three or four groups said 'Yes, Sir' and Greg responded' "Good, so we are making steps in the right direction'. This demonstrates how Greg used the information from the pre-lesson assessment to emphasise questions the students seemed to find difficult.

In the Time-distance graphs lesson, at the end of the first part of the lesson, in which students had matched the graphs with the descriptions, Greg said to the class that it seemed that there was general agreement. He asked if there were any 'big arguments'. He said they were going to do one more thing but first they would go through the solutions and check their matches. He read the answers from the lesson guide. At the end some students said 'Yay!' and Greg asked 'Is everyone happy?'. It seemed they were. In this case the teacher was probably not expecting anyone to be unhappy, as he had already checked that each of the small groups seemed to have understood.

In both the third and fourth lessons Greg had big versions of the small cards and the students who finished early stuck them on the board in groups. Greg led the discussion about the matches.

Perhaps unsurprisingly, there is little evidence of formative assessment taking place in this part of the lesson. It seems that in most of the lessons, certainly in the latter three, that Greg was confident that the students were comfortable with the work and knew what they were doing.

Student engagement

In this section we address questions such as:

- Did the students used the teacher as resource? Did the students use their peers as resource?

As stated, the lessons were designed to engage the students as they were required to match the cards, in their small groups. Our observations, and the reports of both the teacher and the students, suggest that the students were engaged. The student responses to the questionnaire in the second and third lessons highlighted phrases such as 'made us discuss' and 'made me think' which are, in our view, powerful indicators of engagement.

In terms of the students' use of the teacher as a resource, we have already described above that the students did use the teacher as a resource, asking questions when they were uncertain of whether their matches were correct, for example.

Our main focus in the classroom was on the teacher so it was more difficult to see how students used their peers as a resource. However, we do have some data and it seems that peers were used as a resource within the small groups, between small groups and in whole class activity.

We observed that the misconceptions of one student was sometimes revealed by an incorrect matching of a card, and then the other student(s) would explain why it was wrong. More often, however, a student would choose a card, read it out and then suggest where it should be placed, explaining his or her reasoning. In one small group, for example, in the fourth lesson, two boys were working together. The one boy, Rory, picked up and read out the card $g(x) > f(x)$ only when x is positive and placed it under one of the graph cards. He then said 'No… no' and the other boy picked it up. Rory pointed to another graph and said 'here, it's this one'.

Using peers as a resource took place between groups less often, as might be expected because the requirement for groups to share their work was not included in all lessons. However, it was a requirement in the exponents lesson, where we observed first that the students appeared to be a little confused about what they should do, but that some did compare their work with other groups. We observed one girl visiting another group and discussing the work with them for about three minutes. This does not provide evidence that any of them were learning from one another, but it seems likely that they were.

However, in time-distance graphs lesson, when the students were asked to visit other groups to compare answers, we watched one girl, Naledi, who had been in a group with three boys at the back of the classroom. She visited another group and went through her matches. She said that her group had matched card 3 with Card B, but 'You guys got 3 and F. Howcome?' (See Figures 47, 48 and 49).

3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.

Figure 36: Card description 3

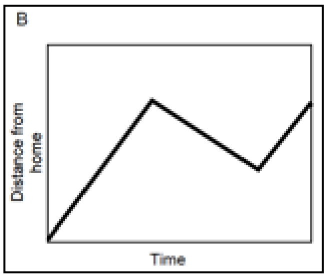


Figure 37: Graph card B

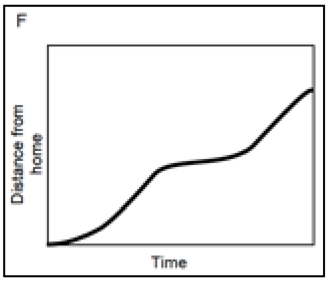


Figure 38: Graph card F

One of the students in the other group went through Card B, explaining that the middle part of the graph showed Tom getting closer to home, not slowing down. It seems that Naledi understood. She went off to visit another group. This suggests that some between these groups peer learning took place.

In the whole class discussions, using peers as a resource was most evident when the students placed the cards on the board and there was some discussion about which cards were placed together. We did not observe the interactions closely but we did notice that there was some discussion and again it seems likely that it was about the mathematics. Hence we could claim that in some way peers were being used as a resource.

The use of technology

In this section we address questions such as:

- How did the teacher/students use the digital resources/technology?

As discussed, the technology we used was mainly card (small cards for the students to match and big cards for the teacher to use on the board). We also used mini whiteboards and Greg used projection technology in some of the lessons.

The use of small cards in matching activities does not necessarily gather information which could inform formative assessment. However, the card matching does provide information which the teacher or peers could use.

We asked Greg on a number of occasions how the cards had helped him. He said that when students were working with them, he could better understand what they were doing and what misconceptions they appeared to have and he used this information to decide what to do next.

The big cards helped support the whole class discussions at the end of the activities. Their role is as a sharing tool.

The mini whiteboards were only used in the first and the last lessons. In both, they were used by the students as a 'working area' to write down their answers or make a calculation, for example. They were not really used in any planned way for formative assessment.

Formative assessment

In this section we address questions such as:

- Which teaching strategies, materials, techniques, … were supportive (or hindering) in terms of the implementation/use of FA and ICT, and with respect to low achievers?

As discussed above, in the first two lessons, Greg used the pre-lesson assessments and this provided him with information about the students' current levels of understanding.

In all the lessons, the main source of information for Greg was the student activity when using the small cards; both the ways in which they worked with the cards and the discussion they had. When he saw that there was some difficulty, he tended to ask questions or provide hints.

We also saw peer formative assessment within the small groups. Students would realise that their peers were not understanding in the same way that they were and would take action accordingly, usually by explaining their thinking.

Concluding comments

- How do you appraise the lesson? (in terms of FA and digital tools). And how does the teacher appraise his/her lesson?

These lessons were carefully designed to provide opportunities for formative assessment although we would tweak the designs if we were to use them again. For example, we did not think the pre-lesson assessments used in the first two lessons added much or were worth the effort. Particularly for the Grade 10 class, Greg knew his students well and already had a good understanding of their strengths and weaknesses. For this reason we did not design pre-assessments for the later lessons. We also thought the introductory activities in the first two lessons did not work very well for Greg and his classes, and again were not really worth the time and effort they took. Again, in the latter two lessons, we cut down on this whole class teaching aspect of the lessons.

It seems that Greg took advantage of the opportunities for formative assessment in a natural way of 'good teaching' but mostly his use of it was unplanned and not deliberate.

Further, most of the lessons were used with a class of high-achieving learners who responded well to taking part in a research project. The first lesson was used with a mixed ability set who did not respond well.

Overall, the lessons were popular with the students and the teacher. He said to us, after all the lessons were done, 'This group enjoyed it because it was novel and I think they saw themselves succeeding (particularly the more able ones). I always got the feeling that, like me, they looked forward to your visits.'

5- Pupil perceptions

Q-sort

On 27 November we visited Fish Hoek High School to administer a Q-sort to nine of the students in Greg Hawtrey's Grade 10 Mathematics class. This Q-sort was developed by the consortium partners and aims to understand something of students' attitudes towards mathematics.

Our visit took place on the last day of the students' exams and these students had volunteered to stay on after school to do the Q-sort. All nine students were white English-speaking from middle class backgrounds. They were mostly good at maths and keen to help.

When we arrived and told the students that they would be working with cards and sticking them on a poster they asked whether they could work in groups. This was possibly because in all the lessons we had done with them they had worked in pairs on an activity that involved sticking cards on a poster. One of the learners later said that she didn't realise that they were staying after school to do an evaluation of the research project. She thought they were staying after school to do a mathematics activity. This might also explain why the students asked to work in groups.

The nine students divided themselves into three groups of three and we handed out the poster paper and cards. I drew four columns on the board and wrote the four headings: Strongly Agree, Agree, Disagree, and Strongly Disagree. We explained to the students that they needed to fold their poster paper into four columns and label them as indicated on the board.

Because the students were working in groups there was a lot of discussion that took place during the Q-sort. When there were statements that a group could not agree on, we told them to put it in the column that most agreed on and to make a note of where the other members of the group would have placed it. Two of the three groups asked me what was meant by 'practical activities' – whether it meant practising mathematics or doing an activity such as measuring the height of a tree. I said it referred to the latter.

To analyse the responses to the Q-sort, we scored each 'strongly agree/disagree' as two or negative two points and each 'agree/disagree' as one or negative one point. Figure 50 shows the scores of the items for which all three groups were in rough agreement, with all three either agreeing or strongly agreeing or, on the other hand, disagreeing or strongly disagreeing.

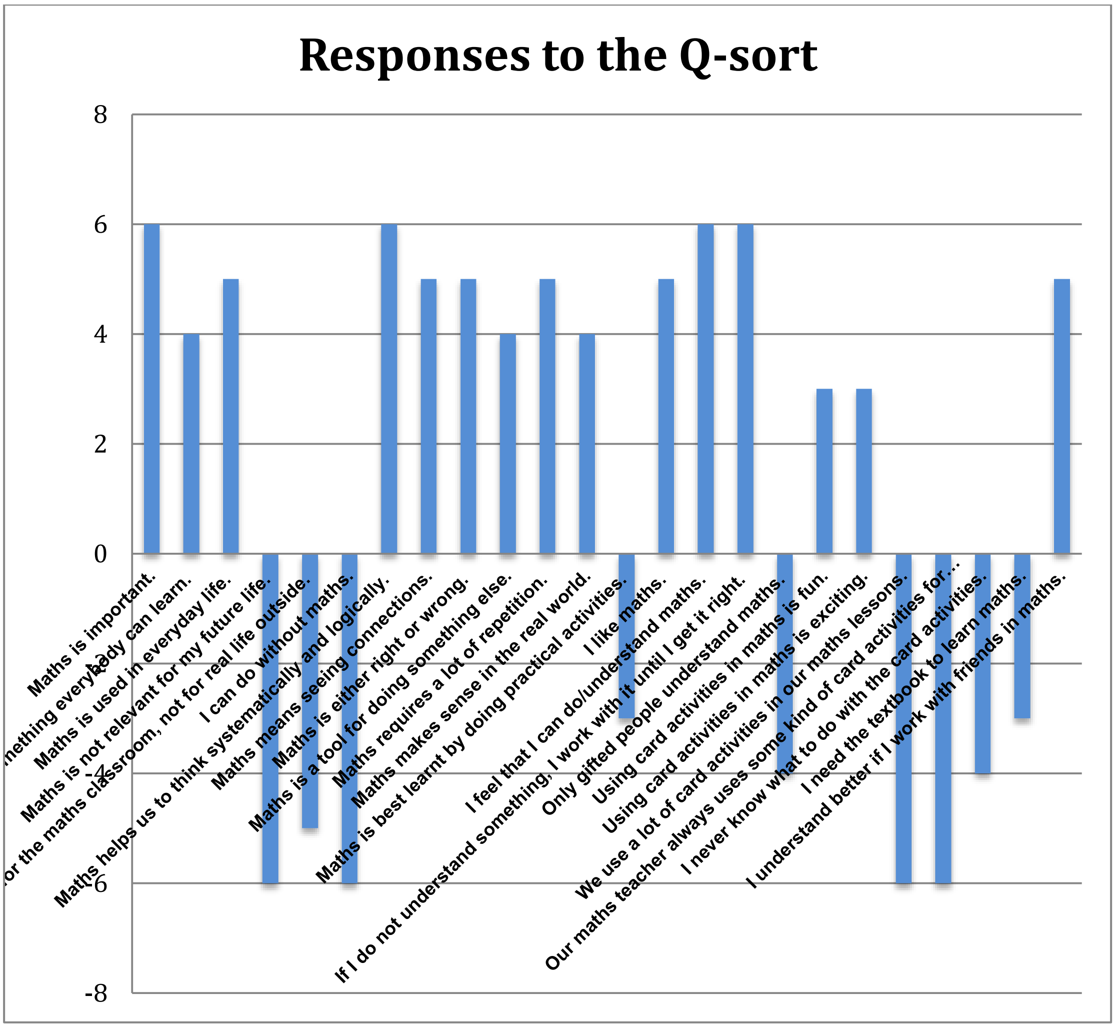


Figure 39: Students' views, the Q-sort

The first set of statements are related to general views on mathematics. It appears that all groups saw mathematics as important and relevant to their lives, with all three groups choosing strongly agree for 'Maths is important' and strongly disagree for 'Maths is not relevant for my future life' and 'I can do without maths'. On why maths is important there was agreement that mathematics helps us think systematically and means seeing connections.

A second set of responses taps into students' own experiences of learning mathematics. There was strong agreement (six points) to the statements 'I feel that I can do/understand mathematics' and 'If I do not understand something, I work with it until I get it right'. The statement 'I like maths' scored five.

A final set of responses related to the use of cards and group work. It seems that there was a mild liking of the card activities, with both 'fun' and 'exciting' scoring three points. There was a general consensus that the teacher does not use card activities in ordinary mathematics lessons (-6 points) but it seems that the students knew what to do (-4 points on 'I never know what to do…'). In terms of group work, there was strong agreement that students understood better if they worked with friend (score of 5).

For the statements for which there was no general agreement, perhaps the most interesting, from the perspective of formative assessment, is the overall score of -3 for the statement 'Our teacher uses the card activities to find out where we are in our learning'.

Questionnaires

For two of the research lessons, we gave the students a questionnaire about the lesson. The analysis of these is given below.

Research lesson 2

First students were asked to select three words or phrases that applied to the activity. The word cloud below (Figure 51) represents their responses.

Figure 40: Choose three words or phrases

Very many of the students (20 out of 25) selected 'group work' and of those who did not select this, four chose different, with a total of 17 choosing 'different' altogether. Fifteen students selected 'made-us-discuss', with a smaller number of ten choosing 'made me think' and 'easy', five choosing 'exciting' and one choosing 'messy'. Of the ten who chose 'easy', however, four also chose 'made us discuss' and two chose 'made me think'.

The students were also asked to complete two sentences: 'The card matching activity was **different** because ....' And 'The card matching activity **would be better if…'.**

For the first of these, ten students gave answers suggesting that this was the first time they had done something like this. For example, one said 'I had never experienced a lesson like this before' and another said 'I have never done anything like that'. Ten referred to working in a group, with some mentioning the need to get everyone in the group to agree or learning from others in the group: 'it forced us to work together', 'the others are there who might see something you might not' and 'together we helped each other'. At least eight comments related to the nature of the task, with one saying that it was like a puzzle and the others referring to the visual, practical or active aspect of the activity. Comments included, for example: 'we learnt through doing the activity … rather than just being told', 'more doing, less listening' and 'we don't use practical things to help us understand maths'. Three students also referred to the different coloured cards, saying that they liked them.

For the second sentence some students' suggestions related to the construction of groups, with eight saying that they thought pair work would be better than groups of four and two saying that it would be better if the teacher constructed the groups.

In terms of practical set up of the task, eight suggested that bigger paper to stick the cards on would be better (our fault as we forgot the A2 sugar paper we had intended to bring along). One suggested that more time should be allowed with six saying that less time should be given. The last group of comments related to the task itself, with five suggesting that the graphs could be more difficult.

The students' responses to the questionnaire fell into three main areas: group work, the learning style and the mathematics involved in the task.

Although it was already known that group work was unusual for this mathematics class, the very high number of students who chose group work as one of the three keywords, and the high number who commented, was perhaps surprising. Although the overall feeling about the group work seems to have been positive, it is perhaps interesting that nearly a third of the students suggested that pair work would be better.

The term 'learning style' refers to the use of the card matching approach, which involves discussing the cards and agreeing the matches. The students' responses to this activity included words such as 'practical' and 'visual'. It could be argued that their descriptions do not really reflect the nature of the activity, but what is perhaps more important is that they were attempting to find a language to do so. They felt it was important to remark on the not-pen-and-paper nature of the task.

In terms of the mathematics, the responses related to 'easy' and 'too much time' were much as expected, as the teacher, Mr Hawtrey, had explained that the students seemed to be comfortable with time-distance graphs before the lesson. However, it seems that the students were still challenged as they said the activity made them think and the overall impression is that they thought it was a worthwhile activity.

Research lesson 3

The questionnaire had been adapted slightly, in the light of previous responses, to include the phrase 'learning from others'. The word cloud given in Figure 52, below, summarises the responses to the first question, which asked learners to choose three words/phrases that stood out for them in relation to the FaSMEd lesson.

Figure 41: Choose three words or phrases

On a sort of meta-level, nine of the 22 respondents selected ‘different’, with 11 choosing ‘exciting’. In terms of the way the lesson was arranged, ten chose ‘group work’ and related to this there were two who said ‘learning from others’ and 12 who selected ‘made us discuss’. Some words relate to the task itself, of which ‘made me think’ (17) stands out most. Four said it was difficult, three said it was confusing and one said it was easy.

The learners were asked to complete the sentence ‘The card matching activity was different because …’. A first set of comments, eight in all, explained that it was different in comparison to normal lessons, with three out of the eight saying that they had never done anything like this before (which raises a question about their perception of the previous FaSMEd lesson). Where they said it was different, some explained how it was different. For example, one said ‘not just listening/exercises’.

In terms of the lesson style, there were eight comments, which referred to it being visual, practical, interactive and physical. One said it was colourful and one said that the design means that you learn by doing.

A final set of comments related to the way the learners reacted to the task (9). Four learners said it was interesting or fun. Others said it was thought provoking, required them to use their common sense, that they had to think outside the box and that they had to apply their previous knowledge. Three said that it introduced them to different graphs and one said that it extended previous knowledge, which would be good for the exams.

The questionnaire finishes with asking learners to complete the sentence 'The card matching activity would be better if …'. Six learners left this blank, and three further students made comments to the effect that it did not need to be improved. One, for example, said 'nothing wrong, it was awesome'.

In terms of the lesson design and implementation, one learner suggested fewer cards and another said there would be 'less small pieces'. One said that there should be no rule cards (this is something we have thought about as well). One said it would be good to have some rough paper to try things out on, and another said the graph cards should be clearer. One suggested that there should be different levels of difficulty within the task. Finally, one suggested that computers should be used.

Some comments related to the way the classroom was organised, with three more time should be given for the task and three suggesting bigger groups. Two others mentioned some unhappiness about their partners. For example, one said 'we got to work with others, which was cool and stuff, but my partner was a slacker.'

A discussion with the class: research lesson 4

After the fourth research lesson, we held a short discussion about the design of the activity. We asked the students first how they had experienced it. We were interested in their views and also wanted them to feel valued. They said, for example:

"I found it hard. But not too hard. We all taught each other rather than the teacher just telling you."

"It was nice to see graphs that we haven't seen. We could answer question about the types of graphs we haven't seen."

Marie went on to discuss some of the detailed design decisions she had made. For example, she explained that her intention had been for S18 [f(x). g(x) > 0 when x is between 0 and 1] to only be matched with G6. The class had also matched it with G2, G4 and G8. She said that one idea would be to change S18 to "ONLY when x is between 0 and 1" so that it could only be matched with G6. One of the learners said that she liked the fact that some of the statement cards went with more than one graph card because it meant you couldn't just stop once you had found a graph card to match it with.

Finally, we asked them if they had any advice about how the lesson could be used with other classes. There were a few suggestions. For example, one of the learners said: "I couldn't remember how to do the formulas so it would have been nice if the teacher had reminded us what to do.” Marie said that perhaps the introduction should make it explicit that it was not necessary to use the formulas in this activity.

While this discussion was mainly about the design of the lesson, it also provides some insight into the students’ perceptions.

Conclusion

This case study was about Greg Hawtrey and mainly his Grade 10 class. Greg is an experienced teacher, but he told us in a response on the questionnaire, that taking part in the project had been ‘like a shot in the arm’. He said that he had really enjoyed it, and he ‘found that being involved stimulating and exciting (and my pupils have found the same)’. He added ‘It has made me realise that learners can benefit from an alternative approach’.

Our final remarks aim to sum up some responses to the research questions:

- How do teachers process formative assessment data from students using a range of technologies?’
- How do teachers inform their future teaching using such data?
- How is formative assessment data used by students to inform their learning trajectories?
- When technology is positioned as a learning tool rather than a data logger for the teacher, what issues does this pose for the teacher in terms of their being able become more informed about student understanding?

Greg’s processing of formative assessment data was, during the course of the interventions, mainly in real-time and resulted in some interventions with small groups, some decisions made for the whole class discussion and some thoughts about what he might do if he were to teach the same lesson again. The main technology he used was small and big cards, although mini whiteboards were also used. He told us, however, that he intends to use mini whiteboards with his classes in the future. In terms of *how* he processed the data, we have little idea, but can speculate that this was informed by his experience, his knowledge of his students and what he observed in the classroom.

Where ‘future’ teaching is mentioned, we have already discussed future teaching within the same lesson. In terms of future teaching with this class, we have no evidence of how data collected during the research lessons might have influenced lesson that took place at a later stage. In terms of teaching the same lesson another time, to a different class, we suggest that formative assessment data (i.e. how the students responded to the activity) would be one of many factors, including the lesson design, that might influence his decisions.

For the students, it seems unlikely that they would be interested in formative assessment data. It appeared that they were very engaged in the tasks we gave them, and they saw their role as completing the task (as they should, (Sierpinska, 2004)). We can say that we saw them using formative assessment from their peers to inform how they worked together with them, but cannot say anything about how they used formative assessment to inform their own learning trajectories.

The technology we used can be seen as a learning tool rather than a data logger. We have discussed how the use of small and big cards, particularly the small cards, gave the teacher valuable information about the students' understanding and we have explained how the teacher used this information.

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